ODD VERTEX ANALYTIC MEAN LABELING OF SNAKE GRAPHS

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ABSTRACT

A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. A graph G with p vertices and q edges is said to have an odd vertex analytic mean labeling if there exists an injective function $f:V(G)\rightarrow\{1,3,5,...,2q-1\}$ such that the induced map $f^*: E(G)\rightarrow\{1,2,...,N\}$ defined by

$$f^*(e=uv)=\begin{cases}\frac{|[f(u)]^2-[f(v)]^2|}{2} & if\ |[f(u)]^2-[f(v)]^2|\ is\ even\\ \frac{|[f(u)]^2-[f(v)]^2|+1}{2} & if\ |[f(u)]^2-[f(v)]^2|\ is\ odd\end{cases}$$

and the edge labels are distinct. A graph that admits an odd vertex analytic mean labeling is called an odd vertex Analytic Mean Graph.

Keywords: Graph labeling, odd vertex analytic mean labeling, odd vertex analytic mean graph.

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1. INTRODUCTION

Throughout this paper, by a graph we mean a finite, undirected, simple graph. Let G(V,E) be a graph with p vertices and q edges. For notations and terminology we follow [2]. The concept of mean labeling was introduced by Somasundaram and Ponraj [5] and analytic mean labeling was introduced by Tharmaraj and Sarasija[6]. Motivated the results in [3] & [6] we introduced a new mean labeling called odd vertex analytic mean labeling. We proved that Triangular Snake graph T_n , Quadrilateral Snake graph Q_n , Double triangular Snake graph $D(T_n)$, Double Quadrilateral Snake graph $D(Q_n)$ are odd vertex analytic mean graphs.

Definition 1.1 A Triangular Snake graph T_n is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i *and* u_{i+1} to a vertex v_i for $1 \le i \le n-1$. That is, every edge of a path is r eplaced by a triangle C_3.

Definition 1.2 A Quadrilateral Snake graph Q_n is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i *and* u_{i+1} to two new vertices v_i *and* w_i. That is, every edge of a path is replaced by a cycle C_4.

Definition 1.3 A Double triangular Snake graph $D(T_n)$ is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i *and* u_{i+1} to a new vertex v_i, w_i, $1 \le i \le n-1$.

Definition 1.4 A Double Quadrilateral Snake graph $D(Q_n)$ is obtained from a path $u_1, u_2, ..., u_n$ by joining each of the vertices u_i *and* u_{i+1}, $1 \le i \le n-1$ to two new vertices v_i *and* w_i and to two new vertices x_i *and* y_i, $1 \le i \le n-1$respectively and adding an edge between each pair of vertices (v_i, x_i) *and* (w_i, y_i).

2. MAIN RESULTS

Theorem 2.1

Triangular Snake graph T_n is an odd vertex analytic mean graph.

Proof:

Let $V = \{u_i \; / 1 \le i \le n\} \cup \{v_i \; / 1 \le i \le n-1\}$ be the vertex set and

$E = \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i \; / 1 \le i \le n-1\} \cup \{u_{i+1} v_i \; / 1 \le i \le n-1\}$ be the edge set of Triangular Snake graph T_n.

Here $|V(G)| = 2n-1$ *and* $|E(G)| = 3n-3$

Define a function $f : V \to \{1,3,5,...2q-1\}$ by

$f(u_i) = 2i-1, \; 1 \le i \le n$

$f(v_i) = 2n+2i-1, \; 1 \le i \le n$

Then the edges are labeled as follows

For $1 \le i \le n-1$

$$f^*(u_i u_{i+1}) = \frac{\left| [f(u_i)]^2 - [f(u_{i+1})]^2 \right|}{2}$$

$$= \frac{\left| [(2i-1)]^2 - [(2i+1)]^2 \right|}{2}$$

$$= 4i$$

For $1 \le i \le n-1$

$$f^*(u_i v_i) = \frac{\left|[f(u_i)]^2 - [f(v_i)]^2\right|}{2}$$

$$= \frac{\left|[(2i-1)]^2 - [(2n+2i-1)]^2\right|}{2}$$

$$= 2n^2 - 2n + 4ni$$

For $1 \le i \le n-1$

$$f^*(u_{i+1} v_i) = \frac{\left|[f(u_{i+1})]^2 - [f(v_i)]^2\right|}{2}$$

$$= \frac{\left|[(2i+1)]^2 - [(2n+2i-1)]^2\right|}{2}$$

$$= 2n^2 - 2n + 4ni - 4i$$

Then the edge labels are distinct.

Hence, the Triangular Snake graph T_n is an odd vertex analytic mean graph.

Example

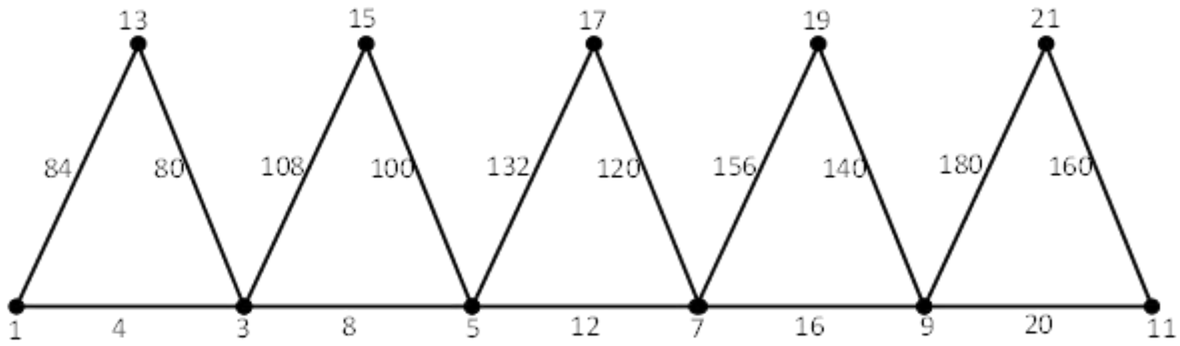

Figure 1. Triangular Snake graph T_6

Theorem 2.2

Quadrilateral Snake graph Q_n is an odd vertex analytic mean graph.

Proof:

Let $V = \{u_i \ / 1 \le i \le n\} \cup \{v_i \ / 1 \le i \le n-1\} \cup \{w_i \ / 1 \le i \le n-1\}$ be the vertex set and

$E = \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i \ / 1 \le i \le n-1\} \cup \{u_{i+1} w_i \ / 1 \le i \le n-1\} \cup \{v_i w_i \ / 1 \le i \le n-1\}$ be the edge set of Quadrilateral Snake graph Q_n.

Here $|V(G)| = 3n-2$ *and* $|E(G)| = 4n-4$

Define a function $f : V \to \{1,3,5,...2q-1\}$ by

$f(u_i) = 2i-1, \ 1 \le i \le n$

$f(v_i) = 2n+4i-3, \ 1 \le i \le n-1$

$f(w_i) = 2n+4i-1, \ 1 \le i \le n-1$

Then the edges are labeled as follows

For $1 \le i \le n-1$

$$f^*(u_i u_{i+1}) = \frac{\left|[f(u_i)]^2 - [f(u_{i+1})]^2\right|}{2}$$

$$= \frac{\left|[(2i-1)]^2 - [(2i+1)]^2\right|}{2}$$

$$= 4i$$

For $1 \le i \le n-1$

$$f^*(u_i v_i) = \frac{\left|[f(u_i)]^2 - [f(v_i)]^2\right|}{2}$$

$$= \frac{\left|[(2i-1)]^2 - [(2n+4i-3)]^2\right|}{2}$$

$$= 2n^2 - 6n + 8ni + 6i^2 - 10i + 4$$

For $1 \le i \le n-1$

$$f^*(u_{i+1} w_i) = \frac{\left|[f(u_{i+1})]^2 - [f(w_i)]^2\right|}{2}$$

$$= \frac{\left|[(2i+1)]^2 - [(2n+4i-1)]^2\right|}{2}$$

$$= 2n^2 - 2n + 8ni + 6i^2 - 6i$$

For $1 \le i \le n-1$

$$f^*(v_i w_i) = \frac{\left|[f(v_i)]^2 - [f(w_i)]^2\right|}{2}$$

$$= \frac{\left|[(2n+4i-3)]^2 - [(2n+4i-1)]^2\right|}{2}$$

$$= 4n + 8i - 4$$

Then the edge labels are distinct.

Hence, the Quadrilateral Snake graph Q_n is an odd vertex analytic mean graph.

Example

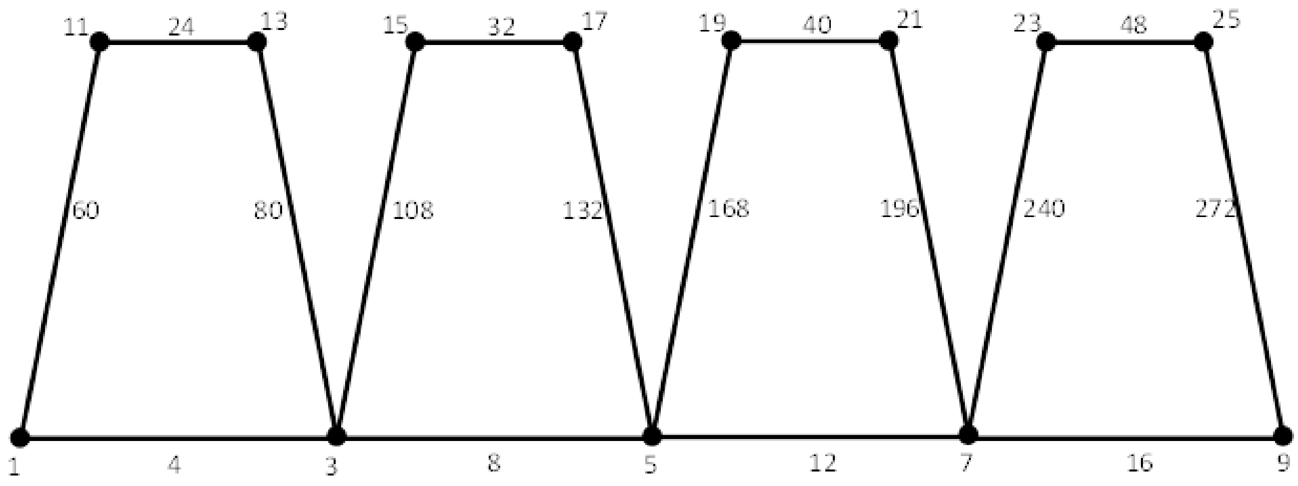

Figure 2. Quadrilateral Snake graph Q_5

Theorem 2.3

The Double Triangular Snake graph $D(T_n)$ is an odd vertex analytic mean graph.

Proof:

Let $V = \{u_i \ /1 \le i \le n\} \cup \{v_i \ /1 \le i \le n-1\} \cup \{w_i \ /1 \le i \le n-1\}$ be the vertex set and

$$E = \{u_i u_{i+1} /1 \le i \le n-1\} \cup \{u_i v_i \ /1 \le i \le n-1\} \cup \{u_{i+1} v_i \ /1 \le i \le n-1\} \cup \{u_i w_i \ /1 \le i \le n-1\} \cup \{u_{i+1} w_i \ /1 \le i \le n-1\}$$

be the edge set of Double Triangular Snake graph $D(T_n)$.

Here $|V(G)| = 3n-2$ *and* $|E(G)| = 5n-5$

Define a function $f : V \to \{1,3,5,...2q-1\}$ by

$$f(u_i) = 2i-1, \ 1 \le i \le n$$
$$f(v_i) = 2n+2i-1, \ 1 \le i \le n-1$$
$$f(w_i) = 4n+2i-3, \ 1 \le i \le n-1$$

Then the edges are labeled as follows

For $1 \le i \le n-1$

$$f^*(u_i u_{i+1}) = \frac{\left|[f(u_i)]^2 - [f(u_{i+1})]^2\right|}{2}$$
$$= \frac{\left|[(2i-1)]^2 - [(2i+1)]^2\right|}{2}$$
$$= 4i$$

For $1 \le i \le n-1$

$$f^*(u_i v_i) = \frac{\left|[f(u_i)]^2 - [f(v_i)]^2\right|}{2}$$
$$= \frac{\left|[(2i-1)]^2 - [(2n+2i-1)]^2\right|}{2}$$
$$= 2n^2 - 2n + 4ni$$

For $1 \le i \le n-1$

$$f^*(u_{i+1} v_i) = \frac{\left|[f(u_{i+1})]^2 - [f(v_i)]^2\right|}{2}$$
$$= \frac{\left|[(2i+1)]^2 - [(2n+2i-1)]^2\right|}{2}$$
$$= 2n^2 - 2n + 4ni - 4i$$

For $1 \le i \le n-1$

$$f^*(u_i w_i) = \frac{\left|[f(u_i)]^2 - [f(w_i)]^2\right|}{2}$$
$$= \frac{\left|[(2i-1)]^2 - [(4n+2i-3)]^2\right|}{2}$$
$$= 8n^2 - 12n + 8ni - 4i + 4$$

For $1 \le i \le n-1$

$$f^*(u_{i+1} w_i) = \frac{\left|[f(u_{i+1})]^2 - [f(w_i)]^2\right|}{2}$$
$$= \frac{\left|[(2i+1)]^2 - [(4n+2i-3)]^2\right|}{2}$$
$$= 8n^2 - 12n + 8ni - 8i + 4$$

Then the edge labels are distinct.

Hence, the Double Triangular Snake graph $D(T_n)$ is an odd vertex analytic mean graph.

Example

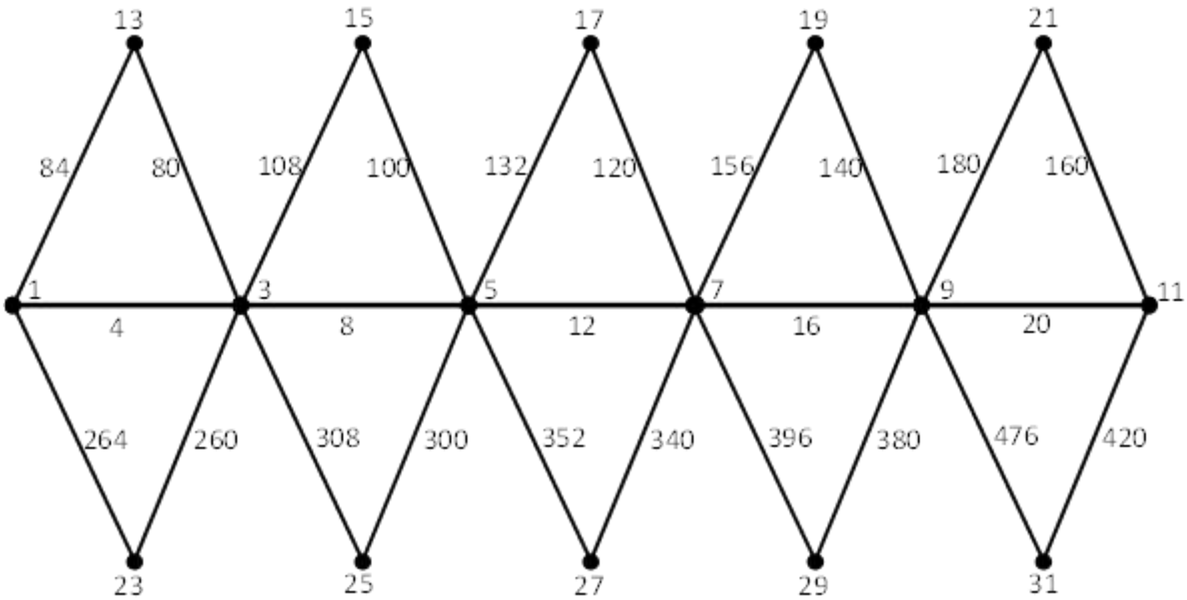

Figure 3. Double Triangular Snake graph $D(T_6)$

Theorem 2.4

The Double Quadrilateral Snake Graph $D(Q_n)$ is an odd vertex analytic mean graph.

Proof:

Let

$V=\{u_i\ /1\le i\le n\}\cup\{v_i\ /1\le i\le n-1\}\cup\{w_i\ /1\le i\le n-1\}\cup\{x_i\ /1\le i\le n-1\}\cup\{y_i\ /1\le i\le n-1\}$ be the vertex set and

$$E=\{u_i u_{i+1}\ /1\le i\le n-1\}\cup\{u_i v_i\ /1\le i\le n-1\}\cup\{u_{i+1}w_i\ /1\le i\le n-1\}$$
$$\cup\{v_i w_i\ /1\le i\le n-1\}\cup\{u_i x_i\ /1\le i\le n-1\}\cup\{u_{i+1}y_i\ /1\le i\le n-1\}\cup\{x_i y_i\ /1\le i\le n-1\}$$

be the edge set of Double Quadrilateral Snake graph $D(Q_n)$.

Here $|V(G)|=5n-4$ *and* $|E(G)|=7n-7$

Define a function $f:V\to\{1,3,5,\ldots 2q-1\}$ by

$$f(u_i)=2i-1,\ 1\le i\le n$$
$$f(v_i)=2n+4i-3,\ 1\le i\le n-1$$
$$f(w_i)=2n+4i-1,\ 1\le i\le n-1$$
$$f(x_i)=4n+4i+1,\ 1\le i\le n-1$$
$$f(y_i)=4n+4i+3,\ 1\le i\le n-1$$

Then the edges are labeled as follows

For $1\le i\le n-1$

$$f^*(u_i u_{i+1}) = \frac{\left|[f(u_i)]^2 - [f(u_{i+1})]^2\right|}{2}$$
$$= \frac{\left|[(2i-1)]^2 - [(2i+1)]^2\right|}{2}$$
$$= 4i$$

For $1 \le i \le n-1$

$$f^*(u_i v_i) = \frac{\left|[f(u_i)]^2 - [f(v_i)]^2\right|}{2}$$
$$= \frac{\left|[(2i-1)]^2 - [(2n+4i-3)]^2\right|}{2}$$
$$= 2n^2 - 6n + 8ni + 6i^2 - 10i + 4$$

For $1 \le i \le n-1$

$$f^*(u_{i+1} w_i) = \frac{\left|[f(u_{i+1})]^2 - [f(w_i)]^2\right|}{2}$$
$$= \frac{\left|[(2i+1)]^2 - [(2n+4i-1)]^2\right|}{2}$$
$$= 2n^2 - 2n + 8ni + 6i^2 - 6i$$

For $1 \le i \le n-1$

$$f^*(v_i w_i) = \frac{\left|[f(v_i)]^2 - [f(w_i)]^2\right|}{2}$$
$$= \frac{\left|[(2n+4i-3)]^2 - [(2n+4i-1)]^2\right|}{2}$$
$$= 4n + 8i - 4$$

For $1 \le i \le n-1$

$$f^*(u_i x_i) = \frac{\left|[f(u_i)]^2 - [f(x_i)]^2\right|}{2}$$
$$= \frac{\left|[(2i-1)]^2 - [(4n+4i+1)]^2\right|}{2}$$
$$= 8n^2 + 4n + 16ni + 6i^2 + 6i$$

For $1 \le i \le n-1$

$$f^*(u_{i+1}y_i) = \frac{\left|[f(u_{i+1})]^2 - [f(y_i)]^2\right|}{2}$$
$$= \frac{\left|[(2i+1)]^2 - [(4n+4i+3)]^2\right|}{2}$$
$$= 8n^2 + 12n + 16ni + 6i^2 + 10i + 4$$

For $1 \le i \le n-1$

$$f^*(x_i y_i) = \frac{\left|[f(x_i)]^2 - [f(y_i)]^2\right|}{2}$$
$$= \frac{\left|[(4n+4i+1)]^2 - [(4n+4i+3)]^2\right|}{2}$$
$$= 8n + 8i + 4$$

Then the edge labels are distinct.

Hence, the Double Quadrilateral Snake graph $D(Q_n)$ is an odd vertex analytic mean graph.

Example

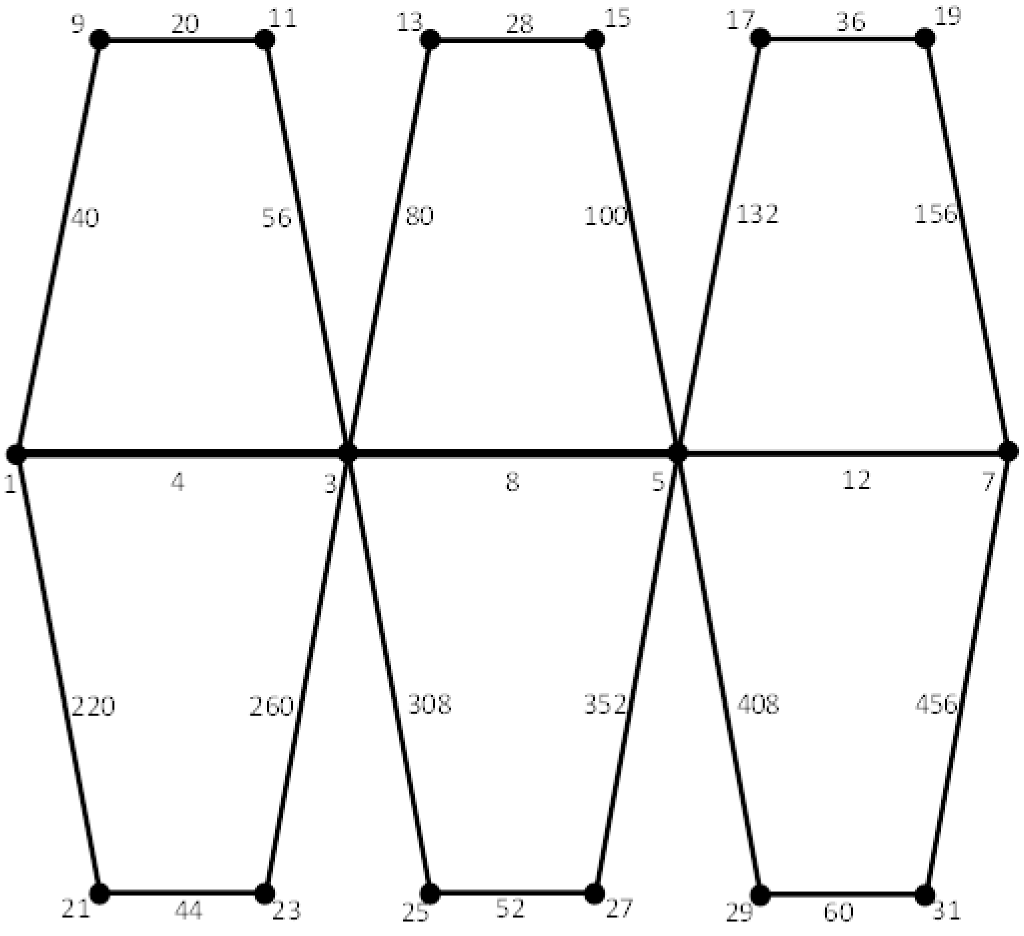

Figure 4. Double Quadrilateral Snake Graph $D(Q_4)$

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