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Stability analysis for a remotely controlled weapon station

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Abstract

A remotely controlled weapon station or automated turret is a two-degree-of-freedom articulated system that supports a weapon to be aimed in a controlled manner. Using the Lagrange equations, a nonlinear model presented in equations of state is proposed, and then a Lyapunov stability analysis is applied and the relevant conclusions are issued.

Index Terms—Turret, 2 DoF System, Stability, Lyapunov.

Introduction

A remotely controlled weapon station or automated turret is a two-degree-of-freedom articulated system that supports a weapon, which after designating a target, is aimed and operated remotely. Actuated by two servo motors to provide lifting and azimuth movements and fed back by their respective encoders, the system is mounted on land or naval vehicles and can be operated during vehicle movement. This displacement can add to the system disturbances associated with the surface on which the vehicle transits, in the case of a warship, the waves and water movements are abstracted with non-linear models.

Fig. 1. Turret design

The present article presents a model of the nonlinear system from the kinematic equations using the Denavit-Hartenberg notation, then the Euler-Lagrange dynamical equations are derived thus generating the system of differential equations that describe the dynamics of the system.

Subsequently, a stability analysis is performed on the system modeled in state variables, with the aim of obtaining a complete description of the model so that in a future work it is facilitated to design an adequate controller and reject the disturbances that may affect the correct functioning of the physical device.

II Dynamic system model

The mathematical modeling of the remote weapon station system is based on a two-degree freedom system of the prototype under development.

The mechanical assembly is divided into two parts: the base platform, which produces the rotational angle on the Z-axis or azimuth, and the lifting turret that provides the elevation angle.

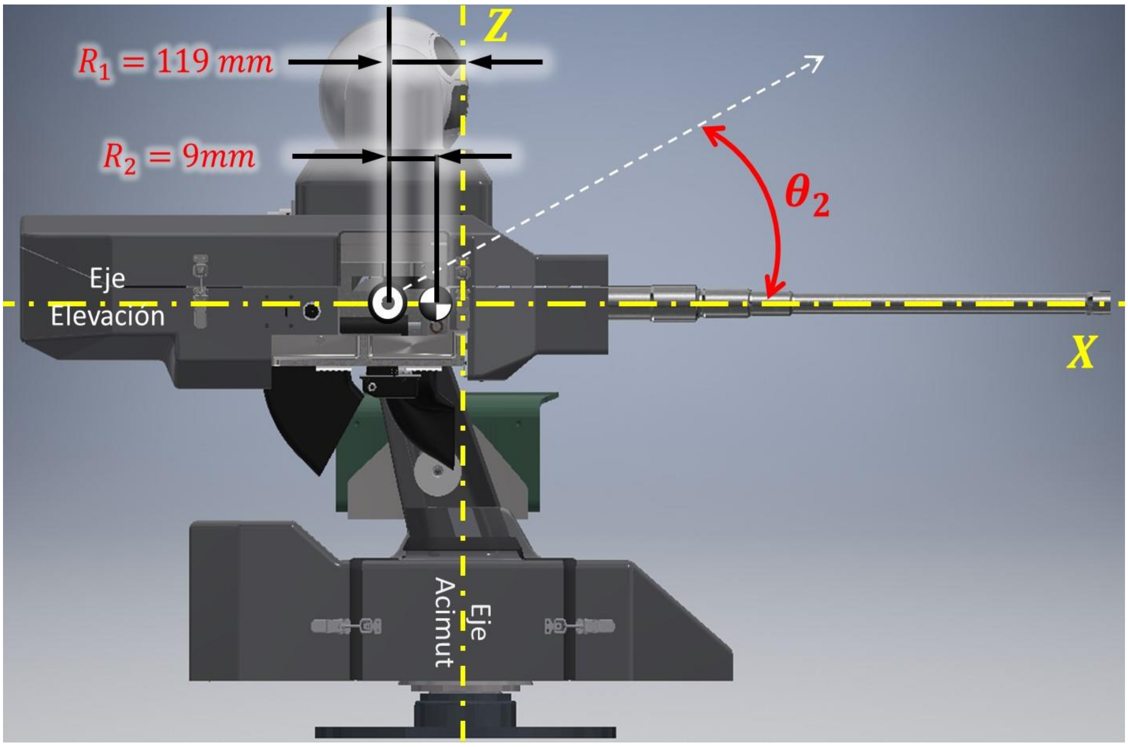

Fig. 2. Side view of the turret

As can be seen in the 2 and 3 the elevation angle will be represented with *θ1* and the azimuth angle with *θ2*. The total mass of the system is m *1*, this means that the complete system will move in azimuth, and *m2* is the mass of the components that rotate in elevation (weapon, camera, ammunition box). The distance between the axes of rotation along the x-axis is *R1* and the length between the axis of rotation and the center of mass of *m2* is *R2*.

To generate the model of the system, the kinematics of the center of gravity of the lifting turret are first found. For this the Denavit-Hartenberg notation is used, and the transformation matrix is found:

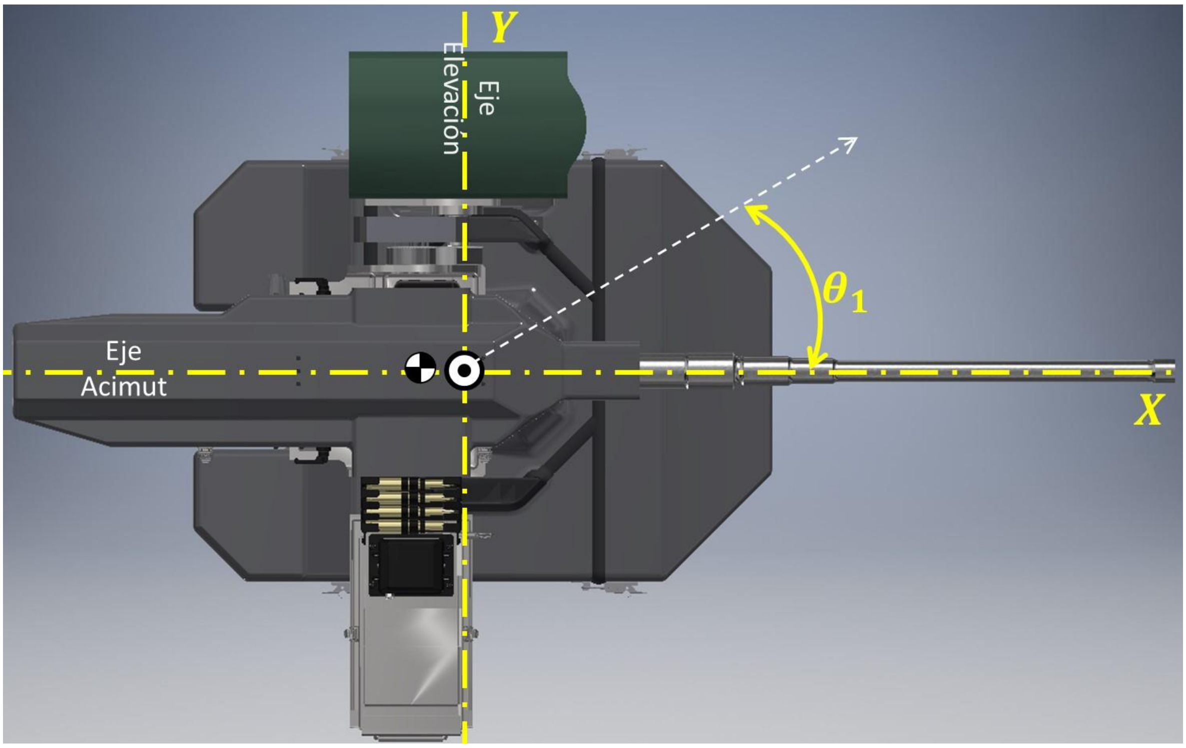

Fig. 3. Top view of the turret

Table 1. *Parámetros Denavit-Hartenberg*

Link	a_i	a_i	d_i	θ_i
1 (base)	R_1	90°	0	θ_1
2 (turret)	R_2	0°	0	θ_2

The compact notation$Sin\theta = S\theta$ and $Cos\theta = C\theta$. The matrix of is:$transformación\ {}_0^2T$

$$ {}_0^2T = \begin{pmatrix} C\theta_1 C\theta_2 & -C\theta_1 S\theta_2 & S\theta_1 & R_1 C\theta_1 + R_2 C\theta_1 C\theta_2 \\ S\theta_1 C\theta_2 & -S\theta_1 S\theta_2 & -C\theta_1 & R_1 S\theta_1 + R_2 S\theta_1 C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & R_2 S\theta_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (1) $$

The X position of the center of mass of set 2 (turret) is given by the fourth column of the matrix of $transformación\ {}_0^2T$

$$ \begin{gathered} X_x = R_1 C\theta_1 + R_2 C\theta_1 C\theta_2 \\ X_y = R_1 S\theta_1 + R_2 S\theta_1 C\theta_2 \\ X_z = R_2 S\theta_2 \end{gathered} \quad (2) $$

To find the dynamic equations of the system, use the Euler-Lagrange equations following the following steps [6]:

- Calculation of the kinetic energy of the system.
- Calculation of the potential energy of the system.
- Lagrangian calculation.

- Solve the equations for each degree of freedom.

Calculation of the kinetic energy of the system

The kinetic energy of the base is calculated as a body that does not present translational motion, only rotation on the horizontal plane. Therefore, its kinetic energy is:

$$K_1 = \frac{1}{2}J_1\omega_1^2 = \frac{1}{2}J_1\dot{\theta}_1^2 \tag{3}$$

The kinetic energy of the barrel array is calculated:

$$K_2 = \frac{1}{2}m_2v_2^2 + \frac{1}{2}J_2\omega_2^2 = \frac{1}{2}m_2v_2^2 + \frac{1}{2}J_2\dot{\theta}_2^2 \tag{4}$$

The total kinetic energy of the system is:

$$K = K_1 + K_2$$
$$K = \frac{1}{2}J_1\dot{\theta}_1^2 + \frac{1}{2}m_2v_1^2 + \frac{1}{2}J_2\dot{\theta}_2^2 \tag{5}$$

where is the linear velocity of the center of mass of the lifting turret and is written in terms of from the position of the center of mass and the derivative in time of its components:$v_2^2\theta_1 y\theta_2$

$$v = \dot{X} = \begin{bmatrix}\dot{X}_x & \dot{X}_y & \dot{X}_z\end{bmatrix}^T$$
$$\dot{X}_x = -R_1\dot{\theta}_1 S\theta_1 + R_2\left(-\dot{\theta}_1 S\theta_1 C\theta_2 - \dot{\theta}_2 S\theta_2 C\theta_1\right)$$
$$\dot{X}_y = R_1\dot{\theta}_1 C\theta_1 + R_2\left(\dot{\theta}_1 C\theta_1 C\theta_2 - \dot{\theta}_2 S\theta_1 S\theta_2\right)$$
$$\dot{X}_z = R_2\dot{\theta}_2 C\theta_2 \tag{6}$$

$$v^2 = (R_1 + R_2C\theta_2)^2\dot{\theta}_1^2 + R_2^2\dot{\theta}_2^2 \tag{7}$$

Substituting in the total kinetic energy:

$$K = \frac{1}{2}J_1\dot{\theta}_1^2 + \frac{1}{2}m_2(R_1 + R_2C\theta_2)^2\dot{\theta}_1^2 + \frac{1}{2}m_2R_2^2\dot{\theta}_2^2 + \frac{1}{2}J_2\dot{\theta}_2^2 \tag{8}$$

Calculation of the potential energy of the system

The movement of the base is restricted to a rotational movement in the horizontal plane, so its center of mass does not present a change in height.

$$U = U_1 + U_2$$
$$U_1 = m_1Gh = m_1G(0) = 0$$
$$U_2 = m_2Gh = m_2GR_2sin\theta_2$$
$$U = U_2 = m_2GR_2sin\theta_2 \tag{9}$$

This system only stores gravitational potential energy in the lifting turret component.

Lagrangian calculus

The Lagrangian is formed from the sum of the total kinetic and potential energies.

$$L = K - U \tag{10}$$
$$L = \frac{1}{2}J_1\dot{\theta}_1^2 + \frac{1}{2}m_2\left((R_1 + R_2C\theta_2)^2\dot{\theta}_1^2 + R_2^2\dot{\theta}_2^2\right) + \frac{1}{2}J_2\dot{\theta}_2^2 - m_2R_2GS\theta_2$$

From this Lagrangian, the system of equations of motion is given by:

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}} - \frac{\partial L}{\partial\theta} = \tau_i \tag{11}$$

Solve the equations for each degree of freedom

$$\frac{\partial L}{\partial\dot{\theta}_1} - \frac{\partial L}{\partial\theta_1} = \tau_1 = (J_1 + m_2(R_1 + R_2C\theta_2)^2)\ddot{\theta}_1 - m_2(R_1 + R_2C\theta_2)^2\dot{\theta}_1 \tag{12}$$

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}_2} - \frac{\partial L}{\partial\theta_2} = \tau_2$$
$$= (m_2R_2^2 + J_2)\ddot{\theta}_2 - m_2R_2S\theta_2(R_1 + R_2C\theta_2)\dot{\theta}_1^2 + m_2R_2GC\theta_2 \tag{13}$$

After developing these steps we get the Lagrange equation:

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + K(\theta) = \tau \tag{14}$$

With

$$M(\theta) = \begin{pmatrix} J_1 + m_2(R_1 + R_2C\theta_2)^2 & 0 \\ 0 & m_2R_2^2 + J_2 \end{pmatrix} \tag{15}$$

$$C(\theta,\dot{\theta}) = \begin{pmatrix} -m_2(R_1 + R_2C\theta_2) & 0 \\ m_2R_2S\theta_2(R_1 + R_2C\theta_2)\dot{\theta}_1 & 0 \end{pmatrix} \tag{16}$$

$$K(\theta) = \begin{pmatrix} 0 \\ m_2GR_2C\theta_2 \end{pmatrix} \tag{17}$$

Clearing the system of equations is:$\ddot{\theta}_i$

$$\ddot{\theta}_1 = -\frac{C_{11}}{M_{11}}\dot{\theta}_1 + \frac{\tau_1}{M_{11}} \tag{18}$$

$$\ddot{\theta}_2 = -\frac{C_{21}}{M_{22}}\dot{\theta}_1 - \frac{K_2}{M_{22}} + \frac{\tau_2}{M_{22}} \tag{19}$$

A change of variables is made to organize the system in state-space representation:

$$x = [\theta_1 \quad \dot{\theta}_1 \quad \theta_2 \quad \dot{\theta}_2]^T \tag{20}$$
$$x_1 = \theta_1 \quad x_2 = \dot{\theta}_1 \quad x_3 = \theta_2 \quad x_4 = \dot{\theta}_2$$

State variables are replaced to form a fourth-order system.

$$\dot{x_1} = x_2 \backslash$$
$$\dot{x_2} = -\frac{C_{11}}{M_{11}} x_2 + \frac{\tau_1}{M_{11}}$$
$$\dot{x_3} = x_4$$
$$\dot{x_4} = -\frac{C_{21}}{M_{22}} x_2 - \frac{K_2}{M_{22}} + \frac{\tau_2}{M_{22}} \tag{21}$$

The system is then written in terms of the *state variables* x:

$$\dot{x_1} = x_2$$
$$\dot{x_2} = -\frac{-m_2(R_1 + R_2 C x_3) x_2}{J_1 + m_2(R_1 + R_2 C x_3)^2} + \frac{\tau_1}{J_1 + m_2(R_1 + R_2 C x_3)^2}$$
$$\dot{x_3} = x_4$$
$$\dot{x_4} = -\frac{m_2 R_2 S x_3 (R_1 + R_2 C x_3) x_2^2}{m_2 R_2^2 + J_2} + \frac{-m_2 G R_2 C x_3 + \tau_2}{m_2 R_2^2 + J_2} \tag{22}$$

Stability analysis

For the autonomous system described in 22, it is proposed to analyze its stability using the Lyapunov stability criterion.

First, system equilibrium points without inputs are sought. So for the condition you have:$\dot{x} = 0$

$$x_1 = 0$$
$$x_2 = 0$$
$$x_3 = \pi/2\,, 3\pi/2 \,\ldots$$
$$x_4 = 0 \tag{23}$$

The variable can have any value and the system will remain in the same state for any future time. Physically it is understood that the mechanical assembly of the base remains in a horizontal plane and at any initial angle will remain the same if it does not receive any stimulus.$x_1 \theta_1$

Existence and uniqueness

To evaluate the stability of the system, the existence and uniqueness of the system must be ensured, that is, the system must be Lipschitz. For the system to be Lipschitz there must exist a constant L such that:$\dot{x} = A(t)x + g(t)$

$$||A(t)|| \leq L \tag{24}$$

Where A(t) The matrix A of the system:

$$A(t) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{C_{11}}{M_{11}} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{C_{21}}{M_{22}} & 0 & 0 \end{pmatrix} \tag{25}$$

The infinite norm for a matrix is:

$$||A||_{\infty} = max_i \Sigma_{j=1}^{n} |a_{ij}| \tag{26}$$

Thus, the infinite norm of matrix A is:

$$||A||_{\infty} = max \left[1, \left|-\frac{C_{11}}{M_{11}}\right|, 1, \left|-\frac{C_{21}}{M_{22}}\right|\right] \tag{27}$$

Analyzing the components of the matrix and taking into account that the term is positive, while the term is negative, that its numerators are always positive, it is concluded that the maximum term is in row 2, therefore, the constant $-C_{11} - C_{21}L$ exists if the system is locally Lipschitz:

$$\left|\frac{m_2(R_1 + R_2Cx_3)x_2}{J_1 + m_2(R_1 + R_2Cx_3)^2}\right| \leq L \tag{28}$$
$$|m_2(R_1 + R_2Cx_3)x_2| \leq L|J_1 + m_2(R_1 + R_2Cx_3)^2|$$

The system can take different values for state variables and the constant x_2, x_3L still exists. That is, for any value between and inequality is met. Since the state variable represents the angular velocity of the weapon lift, an operating value range can be defined for this variable, such that the system can have a local dimension that depends on , with a constant value:$x_3 0 2\pi x_2 a x_2$

$$\left|\frac{m_2(R_1 + R_2Cx_3)x_2}{J_1 + m_2(R_1 + R_2Cx_3)^2}\right| \leq a \tag{29}$$

With this it can be confirmed that the system is locally Lipschitz.

The system is globally Lipschitz if the state variables tend to infinity and the infinite norm of *A* has a limit:

$$\lim_{x\to\infty} ||A|| = \lim_{x\to\infty} \left|\frac{m_2(R_1 + R_2Cx_3)x_2}{J_1 + m_2(R_1 + R_2Cx_3)^2}\right| = \to \infty \tag{30}$$

The variable tends to infinity, therefore system is not globally Lipschitz.x_2

Lyapunov direct method

According to Lyapunov's stability theorem [5], the system is stable at an equilibrium point at the origin if there exists a continuously differentiable function such that:$V(x): D \to R^n$

$$V(0) = 0 \tag{31}$$
$$V(x) > 0 \; en \; D/[0]$$
$$\dot{V}(x) \leq 0 \; en \; D$$

Furthermore if , then the origin is asymptotically stable.$\dot{V}(x) < 0 \; en \; D/[0] x = 0$

We then proceed to look for a suitable Lyapunov function. Remembering that this function must be continuously differentiable and defined in a domain containing the origin, it is proposed to use the energy equation as a Lyapunov candidate function. Using equations 8 and 9 the total energy equation is $E_T$formed:

$$E_T = K + U \tag{32}$$
$$E_T = \frac{1}{2}J_1\dot{\theta}_1^2 + \frac{1}{2}m_2\left((R_1 + R_2C\theta_2)^2\dot{\theta}_1^2 + R_2^2\dot{\theta}_2^2\right) + \frac{1}{2}J_2\dot{\theta}_2^2 + m_2R_2GS\theta_2$$

In terms of the Lyapunov equation is:x

$$V(x) = \frac{1}{2}(J_1 + m_2(R_1 + R_2Cx_3)^2)x_2^2 + \frac{1}{2}(m_2R_2^2 + J_2)x_4^2 + m_2R_2GSx_3 \tag{33}$$

You can check the conditions set in 31:

$$V(0) = 0 \tag{34}$$
$$V(0) = \frac{1}{2}(J_1 + m_2(R_1 + R_2C0)^2)0^2 + \frac{1}{2}(m_2R_2^2 + J_2)0^2 + m_2R_2GS0 = 0$$

$$V(x) > 0 \tag{35}$$
$$V(x) = \frac{1}{2}(J_1 + m_2(R_1 + R_2Cx_3)^2)x_2^2 + \frac{1}{2}(m_2R_2^2 + J_2)x_4^2 + m_2R_2GSx_3 > 0$$

The state variables in quadratic function guarantee that the function is greater than zero. The term of the potential energy remains positive if $R1 > R2$, which is the case even if it were not, would remain positive within the values of .$x_3 = [\pi, 2\pi]$

It is now verified if along the trajectories of is less than or equal to zero.$\dot{V}(x)f(x)$

$$\dot{V}(x) \leq 0 \tag{36}$$
$$\dot{V}(x) = (J_1 + m_2(R_1 + R_2Cx_3)^2)x_2\dot{x}_2 + (m_2R_2^2 + J_2)x_4\dot{x}_4 + m_2R_2GCx_3\dot{x}_3 \leq 0$$

$$\dot{V}(x) = 2m_2R_2Sx_3(R_1 + R_2Cx_3)(-x_2^3 + x_2^2) + m_2R_2Gx_4Cx_3 < 0 \tag{37}$$

The inequality is fulfilled since it is always positive, since for this specific model $(R_1 + R_2Cx_3)R1 > R2$ and the term will always be negative. Again, the term of potential energy establishes a limit at which inequality is ensured since it is less than zero when, however, it is possible that the term of the derivative of kinetic energy is greater than that of potential energy making it less than zero. Under these conditions it can be concluded that the system is asymptotically stable.$(-x_2^3 + x_2^2)x_3 = [\pi/2, 3\pi/2]\dot{V}(x)$

Conclusions

The Denavit-Hartenberg parameters and the Euler-Lagrange formalism allow a dynamic model of the turret to be built quickly and easily, which in this case results in a mathematical model with the rotation angles of each joint and .$\theta_1\theta_2$

The obtained model is rewritten in state variables to facilitate stability analysis. The equilibrium points obtained are easily interpreted as the resting state of the turret.

The method used to perform the stability analysis indicates as a result that the system must have operating conditions to ensure its stability around the equilibrium point. These conditions are operating limits for some variables. The Lipschitz constant L depends directly on the variable that is the speed of rotation of the base of the turret, for the constant L to exist x_2 an operating limit speed must be defined, which makes sense because the physical system cannot allow an infinite speed.

The Lyapunov stability criterion also defines operating limits for this particular system under the conditions set out in equations 35 and 36: and for small velocities since the terms of and also depend on the variables and . The system is asymptotically stable under these conditions, i.e. the system tends to the equilibrium point as time tends to infinity [5]. This gives an indication of a possible controller design suitable for the nonlinear system within a defined region around the equilibrium points.$x_3 = [\pi, 2\pi] x_3 = [\pi/2, 3\pi/2] V(x) \dot{V}(x) x_2 x_4$

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