

Software and Ordinary Differential Equations in Engineering Education

By

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Abstract

The present study aims to identify the types of software used to solve ordinary differential equations in engineering education. The method used is the systematic review of empirical-type articles shown in Scopus, EBSCO, ProQuest and SciELO between 2017 and 2022. The result of the ordinary differential equations is presented in different engineering courses, which are solved using different software. It was concluded that Matlab is the most widely used software to solve engineering problems arising in education modeled by ordinary differential equations.

Keywords: Ordinary differential equations, engineering, software.

Introduction

The advance of technology has allowed the development of computers and software that have been incorporated as a tool in higher education, as in engineering careers, where mathematical problems associated with the specialty of university students are often complex problems that present complicated algebraic calculations, graphs of complicated functions and others, which can not be solved using pencil and paper; it is necessary to use the computer to solve problems that do not always have an analytical solution (Ochkov and Bogomolova, 2015).

In engineering, ordinary differential equations are used to model different career processes, which often cannot be solved by analytical methods, which has led to the development of numerical approximation techniques. In addition, by the 1990s, numerical techniques were discovered that could not be implemented in computers because they were primitive. Also, during the last few years, technology has made it possible to implement numerical techniques and find the solution to ODEs using the software (Brito Santos et al., 2020). According to his curriculum, the engineering student takes mathematics courses where linear algebra, differential equations and other subjects are addressed, where differential equations allow modeling multiple processes, for example population growth, radioactive decay, mass and spring system. The latter is the basis for the study of more complicated systems, such as the damping system of vehicles. Engineering students need to study and understand DEs (Cervantes-Barraza et al., 2020).

Engineering students solve problems applied to reality using ODEs and software, such as students of Telecommunication Systems Engineering, Sound and Image of a university in a course addressed topics such as mathematical models on the consequences of vaccination, modeling the spread of malware in wireless sensor networks, modeling and simulation of the epidemiological behavior of influenza, this activity in the subject has been very satisfactory, both for students and teachers, favors the acquisition of transversal competencies: application and practical thinking; analysis and problem solving; effective communication; teamwork and leadership; lifelong learning and planning and time management; knowledge of contemporary problems (Vidal-Melo and Fusterb-Estruch, 2020).

The theoretical and technical knowledge acquired by future engineers is not enough but also the development of social skills and aptitudes for a suitable professional performance (De Melo Bezerra et al., 2018). Thus, activities were proposed to students that address real problems using ordinary differential equations, software and a methodology to achieve competencies: communication and leadership skills, teamwork and solution of an ordinary differential equation, among others (Tharakan, 2020; Tudon-Martinez et al., 2020; Vidal-Melo and Fusterb-Estruch, 2020).

In this research, a systematic review of software and ordinary differential equations (ODEs) in engineering education is carried out. This article aims to identify the types of software used in the solution of ODEs in engineering education. In this sense, the following research questions arise.

Q1. What types of software have been applied to the teaching and learning ordinary differential equations in engineering?

Q2: Which engineering fields have the most research on the use of software in ordinary differential equations?

Q3: What are the effects of software used in engineering education in terms of the results and findings identified in this literature review?

Material and Methods

The method applied in this study consisted of a systematic review of journal articles indexed in the main databases.

The databases Scopus, EBSCO, ProQuest and SciELO, were searched. The following criteria were used in Scopus: a) the following keywords, both in English and Spanish: "software ecuación diferencial aprendizaje", "software ecuación diferencial de ingeniería", b) all open access, c) the search was performed from 2017 to 2022, d) the type of document was articles, and e) English or Spanish language.

Exclusion criteria: a) duplicity of articles, b) articles that were not in the context of engineering education.

Figure 1: Selection of articles

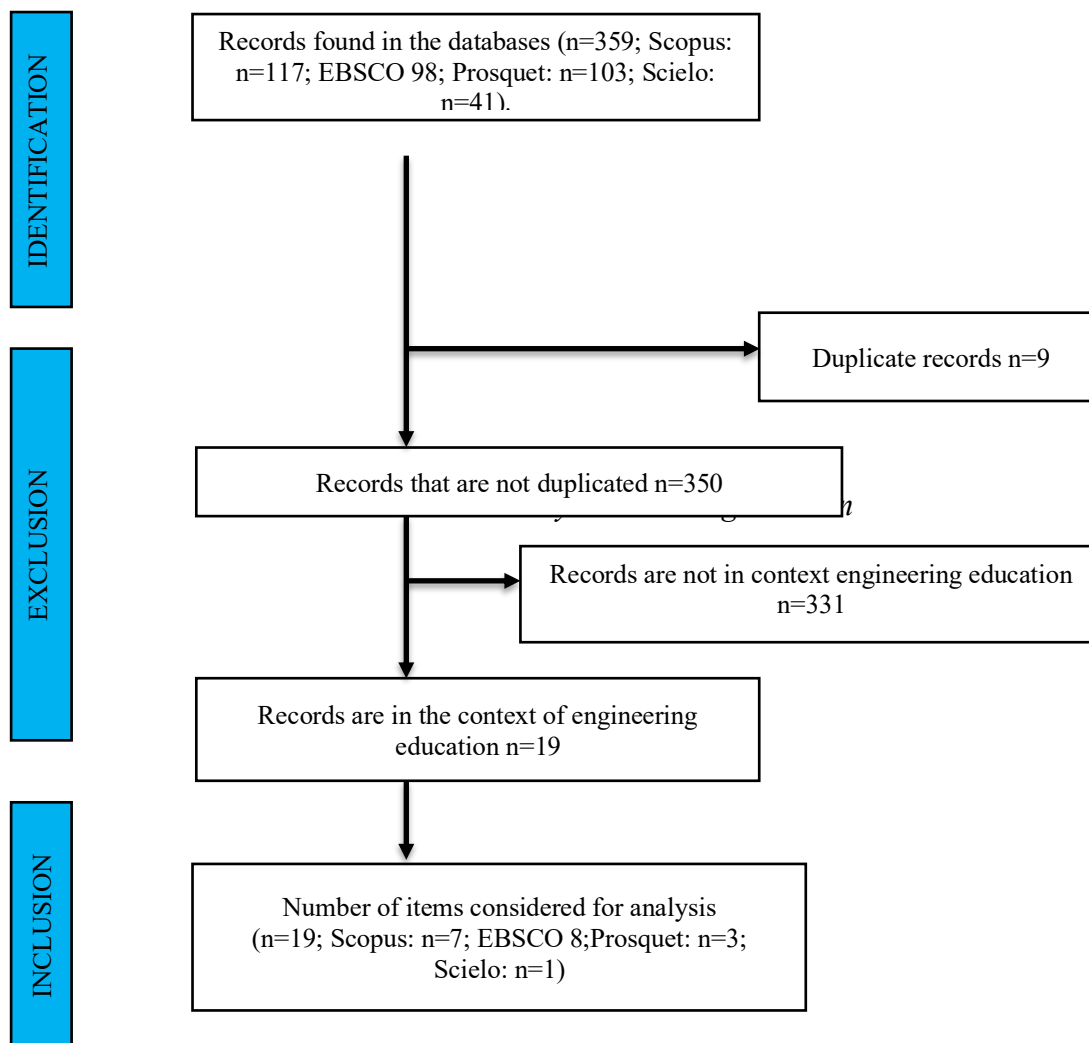


Table 1. Selected items

DATA BASE	Author Year	Research approach	Type of research	Magazine	Country of origin	Objective of the study
SCOPUS	(Hemmerich et al., 2021).	Quantitative	Applied	Engineering in Life Sciences	Germany	Object-oriented modeling of bioprocessing in chemical engineering
	(Tharakan, 2020)	Qualitative	Basic	Journal of Engineering Education Transformations	USA	Develop creative and critical thinking skills to become real-world problem solvers.
	(Roman et al., 2020)	Qualitative	Basic	Journal of Chemical Education	Spain	Implement Monte Carlo method simulations for two case studies of mass transfer processes and compare with

	(Tudon-Martinez et al., 2020).	Quantitative	Applied	International Journal on Interactive Design and Manufacturing	Mexico	deterministic methods using differential equations. Design and implement active learning activities in a virtual environment supported by simulations to teach numerical methods using differential equations. To present a tool according to object-oriented modeling (OOM) developed in MATLAB to create and simulate models, as well as to learn an OOM language and understand how algorithms for partitioning work
	(Jimenez et al., 2019)	Quantitative	Applied	Symmetry	Spain	Present an application of the mathematical theme eigenvalue. Develop an easy to understand and modify Modelica-like system based on the Python programming language. Use elaborated graphical user interfaces (GUIs) in Scilab to enhance learning and solving heat transfer problems in chemical engineering courses.
	(Anderson and McCusker, 2019)	Qualitative	Basic	PRIMUS	Not specified	
	(Kopei et al., 2019)	Qualitative	Basic	PeerJ Computer Science	Ukraine	
EBSCO	(Sérgio Lobato et al., 2021).	Qualitative	Basic	Latin-American Journal of Physics Education	Brazil	

(Develaki, 2019)	Qualitative	basic	Journal of Science Education and Technology	Greece	Explain the nature of computer simulations in science education, and examine certain implications for science education.
(Caicedo and Chacón, 2020).	Qualitative		Praxis & Knowledge	Colombia	Using qualitative methods, design and validate a didactic model for learning differential equations.
(Mendoza Higuera and Cordero Osorio, 2018).	Qualitative	Basica	Latin American Journal of Ethnomathematics	Mexico	Revealing the uses of mathematical knowledge and its resignifications in a community of bionic engineers. Analyze effect of teaching ordinary differential
(Brito Santos et al., 2020).	Quantitative	Applied	International Journal of Engineering Pedagogy	Brazil	equations through a methodology based on PBL, conceptual field theory and Winplot.
(De Las Fuentes-Lara and Aguilar-Salinas, 2020)	Quantitative	Applied	Purposes and Representations	Mexico	To improve students' knowledge, skills and concepts in the topic of spring mass system of second order differential equations .
(Vidal Meló and Estruch Fuster, 2020).	Quantitative	Applied	International Conference on Innovation, Documentation and Education	Spain	To explore other applications of ordinary differential equations, different from the classical problems usually done in class.

PROSQUET	(Jimenez et al., 2019)	Quantitative	Applied	Symmetry	Spain	To present an open source tool according to the object-oriented modeling paradigm and developed in MATLAB. To implement these algorithms
	(Santiago Acosta et al., 2021).	Quantitative	Applied	IOP Publishing	Mexico	To present a proposal of activities for the teaching of mechanical systems modeled by differential equations using Mathematica software.
	(Segarra-Escandón , 2020a)	Qualitative	Basic	Journal of Engineering, Mathematics and Information Sciences	Spain	Perform comparative analysis of numerical methods such as Explicit Euler, Runge Kutta 4 and LocallyExact.
	(Segarra-Escandón , 2020b)	Qualitative	Basic	Journal of Engineering, Mathematics and Information Sciences	Spain	Perform the analysis of Runge-Kutta and Adams Bashforth Moulton numerical methods.
SCIELO	(Cardona et al., 2020)	Qualitative	Basic	University education	Colombia	To present activities implemented in engineering courses for students to operate mathematical modeling processes for understanding physical phenomena using real systems.

Source: Adapted from , pp. 88-91

Table 2. Characteristics of selected items

Author-year	Research subjects	Data collection	Theory used	Software	Conclusions
(Hemmerich et al., 2021).			Object-oriented	Python	Optimization problems can be solved by making use of the

(Tharakan, 2020)	Chemical engineering students	Does not present	Critical thinking	Polymath	<p>pygmo package. In addition, pyFOOMB with the Jupyter notebooks support education in bioprocess engineering and the applied learning of Python as a programming language.</p> <p>-demonstrated that using POLYMATH facilitates and improves students' ability to tackle open problems of ordinary differential equations.</p> <p>- Students developed outstanding and comprehensive reports involving critical and creative thinking to present a broad, open-ended solution to the final project.</p> <p>-Incorporating these open design-type projects, in which students worked in teams, improved their analytical and communication skills. Monte Carlo method simulations can enable understanding and add another dimension to students' computational skills. In addition, the studies shown can enrich the educational competence of teachers. Students acquired knowledge to solve real-life engineering problems.</p>
(Roman et al., 2020)	Does not present	Does not present	Monte Carlo method	Mathcad	<p>-Active learning has emerged as a tool to achieve educational objectives in Science, Technology,</p>
(Tudon-Martinez et al., 2020).	40 engineering students	Questionnaire Homework Examination	Critical thinking Active learning	Matlab	<p>pygmo package. In addition, pyFOOMB with the Jupyter notebooks support education in bioprocess engineering and the applied learning of Python as a programming language.</p> <p>-demonstrated that using POLYMATH facilitates and improves students' ability to tackle open problems of ordinary differential equations.</p> <p>- Students developed outstanding and comprehensive reports involving critical and creative thinking to present a broad, open-ended solution to the final project.</p> <p>-Incorporating these open design-type projects, in which students worked in teams, improved their analytical and communication skills. Monte Carlo method simulations can enable understanding and add another dimension to students' computational skills. In addition, the studies shown can enrich the educational competence of teachers. Students acquired knowledge to solve real-life engineering problems.</p> <p>-Active learning has emerged as a tool to achieve educational objectives in Science, Technology,</p>

(Jimenez et al., 2019)	22 engineering students	Questionnaire Homework	Object-oriented modeling	Matlab	Engineering and Mathematics courses. -Activities should be designed to address topics of their professional career, the use of Matlab software provided students with the opportunity to acquire additional competencies and skills. Use of the tool has had a beneficial effect on learning MOO concepts.
(Anderson and McCusker, 2019)	Does not present	Does not present	Own value Differential equations	Tracker	This hands-on activity enhances the motivation of engineering students and prepares them to model complex systems using linear ordinary differential equations. The proposed approach simplifies the understanding of the system, its modification and improvement, adaptation for other purposes, simplifies the integration into third-party software. The IGU helps teachers with the analysis of chemical problems, in addition this tool allows the simulation in a simplified form of mathematical models very common in chemical engineering, also with it you can work more complicated case studies without the teacher and students have to worry about the methodologies used to solve these problems.
(Kopei et al., 2019)	Does not present	Does not present	Modelica and Python language	Python	
(Sérgio Lobato et al., 2021).	Does not present	Does not present		Scilab	

(Develaki, 2019)	Does not present	Does not present		Fortran	Simulations support reasoning and inquiry activities and skills. In addition to the dynamic and visualized representations it provides, simulation allows for rapid iterations. It thus facilitates the immediate construction and verification of numerous modified or alternative models and their underlying hypotheses. This possibility makes the use of simulations a particularly suitable teaching aid to support contemporary educational approaches, including model-based teaching and nature-oriented curricula in science.
(Caicedo and Chacón, 2020).	5 engineering students	Individual and group activities	Troubleshooting	Maple 18	The didactic model is a tool that encourages activity in differential equations courses and can be replicated in other courses.
(Mendoza Higuera and Cordero Osorio, 2018).				Arduino	The mathematical uses of engineers are not related to the usual mathematics.
(Brito Santos et al., 2020).	47 mechanical engineering students	Test Evaluations Student Statement	Theory of conceptual fields. Problem-based learning	Winplot	The interaction of the students with the proposed problems and Winplot software generates that the concepts taught acquire meaning for the student

(De Las Fuentes-Lara and Aguilar-Salinas, 2020)	34 mechanical engineering students	Worksheets Post-test Diagnostic measuring instruments	Symiotic Representation Theory	Geogebra	-Students conveniently associate the sign of the velocity or slope of the tangent line of the counterweight with its direction. -Difficulty in transitioning from natural language to algebraic language was encountered; they were unable to determine the DE and its initial conditions of position and velocity from the problem. The activity helps students acquire transversal competencies: application and practical thinking; analysis and problem solving; teamwork and leadership; effective communication; among others.
(Vidal-Melo and Fusterb-Estruch, 2020).	20 systems, telecommunications, sound and image engineering students	Survey Examination Group projective	Modeling	Matlab	Use of a Matlab program has had a substantial beneficial effect on learning object-oriented modeling concepts. -The software expands the cognitive capabilities of the user. -The use of dynamic geometry software in the solution of tasks allows the use of new techniques such as dragging.
(Jimenez et al., 2019)	Industrial Electronics and Automatic Control Engineering Students	Questionnaire	Object-oriented modeling	Matlab	The Runge Kutta numerical method of order 4 was identified as having higher Euler Explicit and LocallyExact accuracy.
(Santiago Acosta et al., 2021).	28 university students	Survey	Modeling Reports	Mathematica	
(Segarra-Escandón, 2020a)	Euler Explicit Runge Kutta LocallyExact	Numerical results and graphs	Numerical methods	Mathematica	

(Segarra-Escandón, 2020b)	Runge-Kutta and Adams Bashforth Moulton	Numerical results and graphs	Numerical methods	Mathematica	The Runge Kutta numerical method of order 4 was identified as having higher accuracy than Adams Bashforth Moulton. It is important for future engineers to be familiar with various forms of mathematical modeling so that they are aware of the mistakes that can be made and have sufficient criteria to select the most appropriate technique in their professional work depending on each situation.
(Cardona et al., 2020)	30 engineering students	Problems	Mathematical modeling	Tracker Matlab	

Source: Adapted from , pp. 88-91

Process of compiling the characteristics of the selected items

For the compilation process, a total of nineteen articles are distributed: seven in Scopus, eight in EBSCO, three in ProQuest, and one in SciELO. The information is compiled in two tables. First, Table 1 contains the following fields: database, author and year, focus of the journal of publication, type of research, journal, country of origin, and study objective. Second, Table 2 contains the fields: author-year of publication, research subjects, data collection, the theory used, software and conclusions.

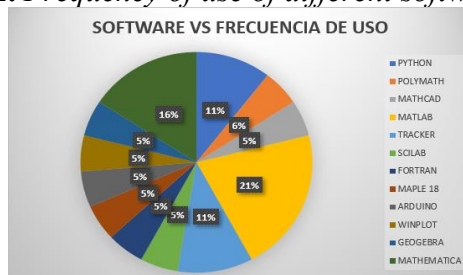
Results

There are few studies addressing software in ordinary differential equations in engineering education; nineteen articles were found from 2017 to 2022, which are found in the databases in Scopus, EBSCO, ProQuest, and SciELO see Table 1.

The nineteen articles chosen to use software: two articles use Python, one Polymath, one Mathcad, four Matlab, two Tracker, one Scilab, one Fortran, one Maple 18, one Arduino, one Winplot, one GeoGebra and three Mathematica, see Table 2.

The research subjects in ten articles are engineering students, and the rest are dedicated to engineering topics, but in all of them, the use of software in differential equations is present.

Figure 2. Frequency of use of different software types



Source: Mathematical Software

Discussion

There is scarce research with free abscess dealing with software in ordinary differential equations in the field of engineering education; this can be observed in the searches performed from 2017 to 2022 in the Scopus, ProQuest and SciELO databases, where only nineteen research articles have been found see Table 1. This is because research in mathematics at a higher level in education is new and not very recurrent.

There is a predominance in Matlab (21%) and Mathematics (16%) software in the research where Matlab is used for modeling, as seen in Figure 2.

The use of students as research subjects prevails. This can be seen in Table 2, in ten articles. Here the software is used to solve differential equations, accompanied by numerical methods when the analytical solution cannot be found.

Conclusions

Matlab is the most widely used software for solving engineering problems arising in education that are modeled by ordinary differential equations.

In engineering, chemical engineering education presents the largest number of research concerning the use of software in differential equations.

The use of the software promotes group work, and technological competence gives the student confidence in the calculations and allows the student to visualize the solutions and give interpretations to the processes they represent.

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