Fisga-Iae Asvsf Algorithm for Effectively Solving the Localization

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Abstract

The use of Smooth Variable Structure Filter (SVSF) has been successfully overcoming the Localization problem. Generally, its performance depends on the knowledge of noise statistics for the process and measurement. Because this knowledge is not available, both are determined and kept to be constant for all iterations. However, this approach will lead SVSF to the divergence condition. Accordingly, a novel improvement, namely FISGA-IAE ASVSF, is proposed in this paper. This name represents the role of the Genetic Algorithm (GA) used to optimize the Fuzzy Inference System (FIS) that is initially applied for enhancing the adaptive SVSF. Unlike the traditional way, this strategy can recursively update the noise covariance of the process Q and measurement R. In detail, FIS supervises the adaptive SVSF to reduce the mismatch between the reference and estimated covariance of error innovation. To effectively arrange the membership function of FIS, the GA is adopted. Lastly, it is implemented to solve the localization problem of mobile robots in the synthetic simulation perception. By using the term RMSE, the comparatively presented results are analyzed. And the proposed method shows better performance in terms of accuracy and stability.

Index Terms: Adaptive SVSF, IAE, FIOS, GA, Localization.

Introduction

The localization is relatively a new problem with an objective of estimating both the spatial and orientation of the mobile robot [1]–[4]. Commonly, the mobile robot is given a knowledge of the real-static map before it is operated [2], [4]–[6]. This map is represented with certain coordinate of all features in the global frame (arena) [7], [8]. The robot is initially placed on the certain position by user and this robot position is informed to it. The robot is equipped with laser scanner and rotary encoder, which are respectively used as the tool to immediately sense the availability of the feature and detect the distance between the previous and recent position after it executes the control command. It cannot be denied that the sensor suffers from the accuracy due to various factors and invisible noise [9], [10]. Regarding to this condition, the process and measurement would always be affected and perturbated. Therefore, in order to determine the accurately estimated values of the robot path, the filtering strategy has been involved [11]–[15]. The popular one is Extended Kalman Filter which utilize the principle of gaussian distribution to recursively update the posterior values of mean and covariance [16]–[18]. However, the use of EKF has been limited because of the factor such as high-cost computation, complexity in calculating the Jacobian Matrix, having a risk of divergence when applied on large dynamic situation [12]. Therefore, different filter is commonly used as its alternative such as Smooth Variable Structure Filter [19]. As the parametric estimator, the SVSF also requires the properly defined covariance matrices of noise statistic which referring to the current system condition [20], [21]. Consequently, keeping them invariant under time integration as the traditional method is not recommended. For this reason, the adaptive filter strategy, such as Innovation Adaptive Estimation, has been popularly involved [14], [22]. However, it suffers from the accurate scale which giving a problem of the mismatch between the theoretical and actual covariance matrix of the innovation error [23]. Therefore, the IAE is still required to be enhanced before it is utilized to improve the estimation performance of SVSF [24]. As presented in this paper, the novel improvement is lies on the presence of Fuzzy Inference System (FIS) that is initially optimized by Genetic Algorithm (GA). Henceforth, it is called FIS-GA-enhanced ASVSF-IAE. It is then used to solve the localization problem of mobile robot. By utilizing the term of RMSE, the estimated values are then compared with the result of the other conventional method. Based on this comparative result the proposed method shows better accuracy and stability.

The rest of this paper is organized as follows: Section II presents the motion model and measurement used for localization. Besides that, the former formulation of SVSF is also presented. Section III presents the concept of proposed method together with the basic theory for each sub-method. Section IV presents the result and analysis. And Section V presents a conclusion

Methodology

A. Kinematic Configuration, Motion Model and Measurement Model

Suppose the robot is placed on the surface environment with x, y as the spatial pose and θ as the orientation or heading of the robot.

$$x_{R,k} = \begin{bmatrix} x_{r,k} \\ y_{r,k} \\ \theta_{r,k} \end{bmatrix} \quad (1)$$

Then the configuration of the robot is modelled as follows

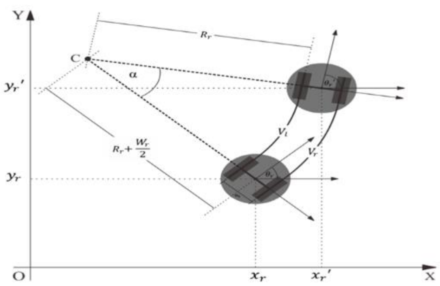

Figure 1 *Kinematic Configuration [14], [22]*

Moreover, by considering that the robot moves and turns based only on the different speed values of the right and left wheel, the following equation represents the kinematic equation for the mobile robot

$$x_R^A = \begin{bmatrix} x_{r,k}^A \\ y_{r,k}^A \\ \theta_{r,k}^A \end{bmatrix} = \begin{bmatrix} x_{r,k-1}^A \\ y_{r,k-1}^A \\ \theta_{r,k-1}^A \end{bmatrix} + v_r \begin{bmatrix} cos(\theta_{r,k-1}^A) \\ sin(\theta_{r,k-1}^A) \\ 0 \end{bmatrix} \quad (2)$$

Equation (2) shows the motion model of the robot when the right and left speed are same after perturbated by random noise. Contrary, when the right and left velocity are not same, in a certain angle the robot will turn depending on this diversity. Therefore, the analogy of this motion can be expressed as follows

$$x_R^B = \begin{bmatrix} x_{r,k}^B \\ y_{r,k}^B \\ \theta_{r,k}^B \end{bmatrix} = \begin{bmatrix} x_{r,k-1}^B \\ y_{r,k-1}^B \\ \theta_{r,k-1}^B \end{bmatrix} + \begin{bmatrix} \left(R + \frac{W}{2}\right)\left(sin\left(\theta_{r,k-1}^B + \alpha\right)\right) + sin(\alpha) \\ \left(R + \frac{W}{2}\right)\left(-cos\left(\theta_{r,k-1}^B + \alpha\right) + cos(\alpha)\right) \\ \alpha \end{bmatrix} \quad (3)$$

Noted that Rand W in (3) are R_r and W_r in Fig. 1. Both of them are the length between the robot's outer wheel and the distance of the separated wheels, respectively. And C is the point of the turn causing angle. Meanwhile, v_r and v_l represent the speed of right and left wheel after following by small perturbation. These velocities are calculated as follows

$$\begin{cases} v_r = \zeta_1 u_r + \zeta_2 (u_r - u_l) \\ v_l = \zeta_1 u_l + \zeta_2 (u_r - u_l) \end{cases} \quad (4)$$

For u_r and u_l are the control command given by the user with a perturbation vector of $\zeta = [\zeta_1\ \zeta_2]^T$ which are the move and turn factor inferencing the motion. Additionally, the second assumption is that the robot is also equipped with a laser with the distance δ_L and bearing β_L are the output of its measurement. Hence, the following model is designed as the model relative to the measurement

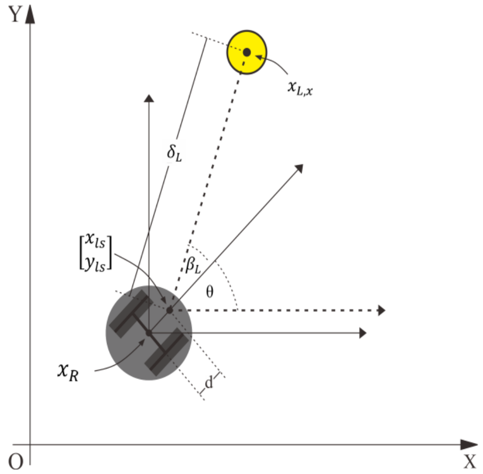

Figure 2 *Feature Detection [14], [22]*

Where d or d_{ls} refers to the laser scanner displacement respect to the robot frame. Meanwhile, $x_{ls}^R = [x_{ls}\ y_{ls}]^T$ refers to the position relative to the robot in local frame, which mathematically calculated as follows

$$\begin{bmatrix} x_{ls} \\ y_{ls} \end{bmatrix} = \begin{bmatrix} x_r \\ y_r \end{bmatrix} + d_{ls} \begin{bmatrix} cos(\theta_r) \\ sin(\theta_r) \end{bmatrix} \quad (5)$$

Continuously, by knowing the robot pose x_R, and the feature coordinate in the global frame $x_L = [x_{L,x}\ x_{L,y}]^T$, the following equation is presented as a direct-based measurement model for a single measurement z_i.

$$\begin{bmatrix} \delta_k^i \\ \beta_k^i \end{bmatrix} = \begin{bmatrix} \sqrt{\left(x_{l,k}^i - x_{ls,k}\right)^2 + \left(y_{l,k}^i - y_{ls,k}\right)^2} \\ atan2\left(\frac{y_{l,k}^i - y_{ls,k}}{x_{l,k}^i - x_{ls,k}}\right) - \theta_{r,k} \end{bmatrix} \quad (6)$$

Naturally, the measurement is noisy, therefore, in order to realistically start the synthetic-based experiment using this model, every measurement z_i is assumed to be perturbated as well. Therefore, the following equation is used to apply the distance and bearing noise $r = [r_\delta\ r_\beta]^T$ to the laser scanner's output.

$$z_i = \begin{bmatrix} \delta_L^i \\ \beta_L^i \end{bmatrix} + \begin{bmatrix} r_\delta \\ r_\beta \end{bmatrix} \quad (7)$$

B. General Formulation of SVSF

The SVSF is relatively new robust estimator, which utilizes the sliding mode concept as the core to tune the gain [25], [26]. Different from the optimal method, Extended Kalman Filter, the SVSF can also be applied for linear and nonlinear system [25]. Besides that, the covariance update form is originally derived without any reduction and simplification [11]. Therefore, the SVSF has been approved and accepted as the proper method to model the uncertainty and remove the error following the real state. In order to ease our understanding, the general form of SVSF is chained in this section, in which it is a revised form indicated by the existence of covariance update equation. Given the dynamic system model as shown below

$$\begin{cases} x_k = f(x_{k-1}, u_{k-1}) + \omega_k \\ z_k = h(x_k) + v_k \end{cases} \quad (8)$$

Where k is the discrete time index, $x \in R^n$ is the state vector, u is the control vector, and $z \in \mathbb{R}^m$ is measurement vector. Additionally, ω and ν are the small additive noises of the process and measurement, respectively, in which their corresponding covariances are denoted by Q and R, respectively. Furthermore, $f(.)$ and $h(.)$ refer to the state transition and measurement function, respectively. It is assumed that the characteristic of this dynamic system model is expressed as follows.

$$\begin{cases} E[\omega_k] = q_k, Cov[\omega_k, \omega_j] = Q_k \delta_{kj}, \\ E[v_k] = r_k, Cov[v_k, v_j] = R_k \delta_{kj}, \\ E[\omega_k, v_j] = 0 \end{cases} \quad (9)$$

Where δ is Kronecker delta function. Whereas, $E[.]$ and $Cov[\ ,\ ]$ represent mean and covariance term, respectively. This equation shows that the noises are uncorrelated and zero-mean with Q and R covariances [23]. As the common filtering method, the SVSF consists of two steps, the time update or prediction stage and measurement update or update stage. Predefining the initial state and covariance with given the control command, the predicted state of SVSF can be calculated as follows

$$\hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1}, u_k) \quad (10)$$

Where $f(.)$refers to the state transition function with F as its corresponding Jacobian matrix. It can be calculated by taking partial derivative of its own function with respect to the state. Then by utilizing it, the predicted state covariance can be computed as expressed as follows

$$P_{k|k-1} = FP_{k-1|k-1}F^T + Q_k \tag{11}$$

Up to this point, the prediction stage is conducted. Furthermore, the measurement update can be sequentially done as follows

$$\hat{z}_{k|k-1} = h(\hat{x}_{k|k-1}) \tag{12}$$

Where $h(.)$ refers to the measurement function with H is considered as its Jacobian form calculated by taking partial derivative its function with respect to the state. Once it is calculated, the innovation sequence can be computed before it is used to determine the online gain of SVSF. Mathematically, it is described as follows

$$e_{z,k|k-1} = z_k - \hat{z}_{k|k-1} \tag{13}$$

Where z_k refers to the real measurement. Theoretically, this innovation sequence has the covariance which can be calculated using the Jacobian matrix of H as presented below

$$S_k = HP_{k|k-1}H^T + R_k \tag{14}$$

Therefore, the gain of SVSF can be determined as

$$K_k^{SVSF} = H^+ \left\{ \overline{\overline{A}} \circ sat\left(\frac{\overline{\overline{e_{z_k|k-1}}}}{\psi_i} \right) \right\} \overline{\overline{e_{z,k|k-1}}}^{-1} \tag{15}$$

Where $.^+$, $\overline{\overline{\ .\ }}$, and $sat(.)$ illustrate to the pseudo inverse, diagonal matrix, and saturation function, respectively. Meanwhile, all the variables used in updating the gain can be sequentially calculated as follows

$$A = \left(|e_{z,k|k-1}|_{abs} + \gamma |e_{z,k-1|k-1}|_{abs} \right) \tag{16}$$

$$\psi = \left(\overline{\overline{A}}^{-1} HP_{k|k-1}H^T S_k^{-1} \right)^{-1} \tag{17}$$

$$sat\left(\frac{\overline{\overline{e_{z_k|k-1}}}}{\psi_i}\right) = \begin{cases} +1 & \frac{\overline{\overline{e_{z_k|k-1}}}}{\psi_i} \geq 1 \\ \frac{\overline{\overline{e_{z_k|k-1}}}}{\psi_i} \quad if & -1 < \frac{\overline{\overline{e_{z_k|k-1}}}}{\psi_i} < 1 \\ -1 & \frac{\overline{\overline{e_{z_k|k-1}}}}{\psi_i} \leq -1 \end{cases} \tag{18}$$

Then, by using the gain of SVSF, the updated state and its corresponding covariance can be determined as presented as follows, respectively.

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k^{SVSF} \tag{19}$$

$$P_{k|k} = (I - K_k^{SVSF}H)P_{k|k-1}(I - K_k^{SVSF}H)^T + K_k^{SVSF}R_kK_k^{SVSFT} \tag{20}$$

It is noted that (1)-(13) are used as the single cycle of the SVSF process. Therefore, in order to roll into the next process, it requires the residual/posteriori error, which can be calculated as follows

$$\hat{z}_{k|k} = h(\hat{x}_{k|k}) \tag{21}$$

Where the relative measurement value of $\hat{z}_{k|k}$ is calculated by referring to the following equation

$$e_{z,k|k} = z_k - \hat{z}_{k|k} \tag{22}$$

As a note, the stability of SVSF's conventional form can be evaluated by adopting the following equation

$$|e_{z,k|k}|_{abs} < |e_{k-1|k-1}|_{abs} \quad (23)$$

The SVSF-Based Localization Principle and Algorithm

In order to start the synthetic-based experiment as the validation for the proposed method in this paper, the following hypotheses are presented:

1. All the features sensed and detected by the robot are assumed as the 2D-represented point feature.
a. The feature is static and does not have any effect to the motion of the robot.
b. The uncertainties caused by the small perturbation are always following the process and measurement, which are normally distributed (Gaussian)
c. The robot knows the environment condition (all the features are assumed to be available). And all the distinctive features are spread in this environment without any ambiguous.
d. The correspondence is known and the robot only can sense a point of feature.
e. Initially robot is placed on a certain position in the global frame with less uncertainty.

Sequentially, the steps are chained to ease the understanding the algorithm of localization based on SVSF. These steps include the prediction, correction, and estimation which are detailly presented as follows

Initialization

Initialize the robot state $\hat{x}_0$ and corresponding covariance P_0 and define Q and R together withγ.

Prediction Step

Compute the predicted state of the robot using (10), in which f(.) is the function representing the motion model in (2) and (3). Next, compute the compute the covariance relative to the predicted state using (11) for

$$F_s = \begin{bmatrix} \frac{\partial x_{r,k}}{\partial x_{r,k-1}} & \frac{\partial x_{r,k}}{\partial y_{r,k-1}} & \frac{\partial x_{r,k}}{\partial \theta_{r,k-1}} \\ \frac{\partial y_{r,k}}{\partial x_{r,k-1}} & \frac{\partial y_{r,k}}{\partial y_{r,k-1}} & \frac{\partial y_{r,k}}{\partial \theta_{r,k-1}} \\ \frac{\partial \theta_{r,k}}{\partial x_{r,k-1}} & \frac{\partial \theta_{r,k}}{\partial y_{r,k-1}} & \frac{\partial \theta_{r,k}}{\partial \theta_{r,k-1}} \end{bmatrix} \quad (24)$$

Equation is the used Jacobian matrix, which is randomly chosen according the diversity of right and left perturbated velocity. Therefore, (24) can possibly having the following characteristic depend on A and B situation, respectively.

$$F_s^A = \begin{bmatrix} 1 & 0 & \left(R+\frac{W}{2}\right)\left(cos(\theta_{r,k-1}+\alpha) - cos(\theta_{r,k-1})\right) \\ 0 & 1 & \left(R+\frac{W}{2}\right)\left(sin(\theta_{r,k-1}+\alpha) - sin(\theta_{r,k-1})\right) \\ 0 & 0 & 1 \end{bmatrix} \quad (25)$$

$$F_s^B = \begin{bmatrix} 1 & 0 & -u_l \,.\, sin(\theta_{r,k-1}) \\ 0 & 1 & u_l \,.\, cos(\theta_{r,k-1}) \\ 0 & 0 & 1 \end{bmatrix} \quad (26)$$

It is noted that u in (26) represent the input command with zero-perturbation.

Innovation Sequence and Update Gain of SVSF

Compute the innovation sequence using (13) after applying the predicted state into h(.) function which represents the measurement model in (7). As a reminder that the correspondence is known. Compute the gain of SVSF using (15) which is firstly initiated by calculating the theoretical covariance of innovation sequence in (14) with

$$H = \frac{\partial z_k^i}{\partial x_{R,k}} = \begin{bmatrix} \frac{\partial \delta_k^i}{\partial x_{r,k}} & \frac{\partial \delta_k^i}{\partial y_{r,k}} & \frac{\partial \delta_k^i}{\partial \theta_{r,k}} \\ \frac{\partial \beta_k^i}{\partial x_{r,k}} & \frac{\partial \beta_k^i}{\partial y_{r,k}} & \frac{\partial \beta_k^i}{\partial \theta_{r,k}} \end{bmatrix} \quad (27)$$

is considered as the Jacobian matrix of H. Detailed it can be presented as follows

$$H = \begin{bmatrix} \frac{-\Delta x}{\sqrt{q}} & \frac{-\Delta y}{\sqrt{q}} & \frac{d_{ls}}{\sqrt{q}}(\Delta x sin(\theta_{r,k}) - \Delta y cos(\theta_{r,k})) \\ \frac{\Delta y}{q} & \frac{-\Delta x}{q} & \frac{-d_{ls}}{q}(\Delta x cos(\theta_{r,k}) + \Delta y sin(\theta_{r,k})) - 1 \end{bmatrix} \quad (28)$$

Where all variable in (28) can be expanded as follows

$$\Delta x = x_{l,k} - x_{ls} \quad (29)$$

$$\Delta y = y_{l,k} - y_{ls} \quad (30)$$

$$q = (x_{l,k} - x_{ls})^2 + (y_{l,k} - y_{ls})^2 \quad (31)$$

Update the state and Covariance

Compute the updated state and its corresponding covariance using (16) and (17), respectively.

It is noted four steps are applied for the time transition of $k-1$ to k. Therefore, once (28)-(31) are obtained, they are used for the next cycle together the residual value calculated using (22).

The Proposed Method

As know that the SVSF requires accurate noise statistic which is actually unknown in the real application. For this reason, the adaptive filtering based on IAE is implemented aiming to enhance the SVSF by completing it with an ability to recursively update the covariance matrices of the process noise Q and measurement noise R.

IAE for SVSF

According to our latest achievement as per [14], the formulation of adaptive SVSF based on IAE is presented. It is gained based on the principle of Maximum Likelihood Estimation (MLE) that is used to derive the covariance of innovation sequence through the state covariance directly. Mathematically, it is expressed as follows

$$\hat{C}_k = \sum_{j=k-N+1}^{k} d_j d_j^T \quad (32)$$

Where d refers to the innovation sequence of SVSF in (22) and sigma notation with a certain limit is a representation of moving window where N is the size of its window. And $\hat{C}$ is called as practical covariance noise of innovation sequence. And according to its values, both the estimated covariance matrix of process noise $\widehat{Q}$ and $\widehat{R}$ are respectively given as follows

$$\hat{Q} = K_k^{SVSF} C_k K_k^{SVSF^T} \quad (33)$$

$$\hat{R} = C_k - HP_{k|k-1}H^T \quad (34)$$

As known that the use of IAE cannot accurately reflect the reality of system and measurement noise due to the presence of a moving window used to calculate the estimated covariance of innovation sequence. Therefore, in order to prevent unexpected occurrence, the use of controller based on Fuzzy Inference System is involved.

FIS for ASVSF based on IAE

The discrepancies between the actual and theorical covariance of innovation sequence can be occurred once the estimated covariance matrices do not reflect the real condition of the system. Therefore, the use of FIS aims to find an approach that can be used to readjust the posteriori noise covariance based on a defined metric representing the mismatch between two different types of innovation sequence covariance. This metric is mathematically expressed as follows

$$\mathrm{DoM} = \mathrm{S_k} - \mathrm{C_k} \qquad (35)$$

Where DoM stands for Degree of Matching. As its aim, DoM is then used to find the adjuster to reupdate the estimated covariance of the process noise in (32) and measurement in (33) from the perception of FIS. Analogically, when the DoM is around zero, the discrepancy is almost none. Therefore, it is no need to change the estimated covariance noise statistic. Contrary, when it shows large or small discrepancy, the adjuster is determined to be used rescale those covariances. Referring to (34), the large discrepancy is found when DoM greater than zero and the small discrepancy is found when DoM is smaller than zero. If the role of adjuster Adj used to readjust, then $\widehat{\mathrm{R}}$ is expressed as follows

$$\widehat{\mathrm{R}}_{\mathrm{new}} = \widehat{\mathrm{R}}_{\mathrm{initial}} + \mathrm{Adj} \qquad (36)$$

Therefore, the adaptation procedure can be presented as follows

If $\mathrm{DoM} \cong 0$, **then** $\mathrm{Adj} = 0$ and $\widehat{\mathrm{R}}_{\mathrm{new}} = \widehat{\mathrm{R}}_{\mathrm{initial}}$
If $\mathrm{DoM} > 0$, **then** Adj is increased and $\widehat{\mathrm{R}}_{\mathrm{new}}$ is decreased based on (36)
If $\mathrm{DoM} < 0$, **then** Adj is decreased and $\widehat{\mathrm{R}}_{\mathrm{new}}$ is increased based on (36)

Meanwhile, when the adjuster Adj is used to redefine $\widehat{\mathrm{Q}}$, its mathematical expression is presented as follows

$$\widehat{\mathrm{Q}}_{\mathrm{new}} = \widehat{\mathrm{Q}}_{\mathrm{initial}} * \mathrm{Adj} \qquad (37)$$

Therefore, the adaptation procedure can be presented as follows
If $\mathrm{DoM} \cong 0$**then** Adj =identity matrix and $\widehat{\mathrm{Q}}_{\mathrm{new}} = \widehat{\mathrm{Q}}_{\mathrm{initial}}$
If$\mathrm{DoM} > 0$, **then** Adj is increased and $\widehat{\mathrm{Q}}_{\mathrm{new}}$ is decreased based on (37)
If$\mathrm{DoM} < 0$, **then** Adj is decreased and $\widehat{\mathrm{Q}}_{\mathrm{new}}$ is increased based on (37)

It is noted that the adjuster Adj is given by FIS with the DoMas the input. Meanwhile, the idea of relationship between $\widehat{\mathrm{Q}}$ and Adj can be explained as shown below. By definition, the theoretical covariance of innovation can be expanded as follows

$$\mathrm{S_k} = \mathrm{H}\left(\mathrm{FP_{k-1|k-1}F^T} + \mathrm{Q}\right)\mathrm{H^T} + \mathrm{R} \qquad (38)$$

Therefore, it is clear to declare that Q gives linear effect to the value ofS. If Q is small than S is decreased and vise-versa. Accordingly, if the discrepancy occurs indicated by the DoM value, then the update procedure of $\widehat{\mathrm{Q}}$ can be done by augmenting or diminishing $\widehat{\mathrm{Q}}_{\mathrm{initial}}$

As known that this adaptation is a separative manner which needs to keep $\widehat{\mathrm{Q}}$ constant when applying an adaptation to $\widehat{\mathrm{R}}$ and vise-versa. Furthermore, the rule used in FIS is depicted as follows.

1. If (DoM is Negative) then (Adj is IncreaseMore) (1)
2. If (DoM is Negative) then (Adj is Increase) (1)
3. If (DoM is Negative) then (Adj is Increase) (1)
4. If (DoM is Zero) then (Adj is Zero) (1)
5. If (DoM is Zero) then (Adj is Zero) (1)
6. If (DoM is Zero) then (Adj is Zero) (1)
7. If (DoM is Possitive) then (Adj is DecreaseMore) (1)
8. If (DoM is Possitive) then (Adj is Decrease) (1)
9. If (DoM is Possitive) then (Adj is Decrease) (1)

Figure 3 *Rule-Base*

Fig. 3 shows the relationship "**if-and**" between the input (three different classification) and output (five different classification) in the FIS with the same weight of "1". The membership of the normal input (before optimization) is presented as follows. Besides that, Fig. 3 also emphasizes that this experiment applies single input single output, three different memberships of DoM as the input corresponding to five different memberships of the Adj. In which, the input membership function is designed as follows

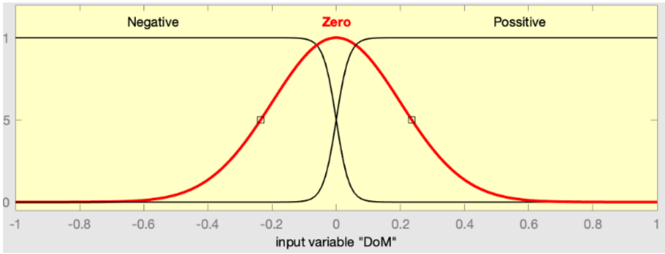

Figure 4 *Input Membership Function*

Based on Fig. 4, the DoM is linguistically divided into three classification of membership function which are "negative" (when it is smaller than zero), zero (when the mismatch around zero), and "positive" (when it is greater than zero). It is considered as the input in this experiment with the adjuster Adj is considered as the output. In which, the membership function of the output is represented by the following figure.

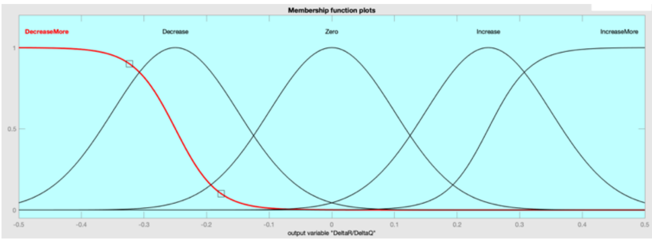

Figure 5 *Output Membership Function*

Fig. 5 depicts that the use of more classification aims to get smooth resolution of the adjusterAdj. Although, all the procedure of FIS is completely presented, as known that the manual setting of membership function is not recommended. It is because there is no available clue representing the need of output when the inputs give a certain value. Therefore, the risk of output dissonance relative to the input and system condition is still high.

Genetic Algorithm used for FIS-enhanced ASVSF-IAE

Based on the reason stated previously, it is proposed an optimization method used to tune the arrangement process of membership function. The idea behind this optimization is to define an arrangement resulting smaller RMSE through the input membership function. The analogy is clear since the mismatch causes large RMSE because of improper adjustment of Q and R makes the filter giving bad condition to the estimated value, then the cause is required to be reduced. Therefore, GA is used to optimize the arrangement of input membership function forDoM. In order to performance GA to FIS-IAE ASVSF, the following steps are considered

- Generating Initial Population
  - Fitness Evaluation
  - Selection
  - Crossover and Mutation
  - Store Generated New Population

Where the fitness function used in this experiment in assumed as RMSE calculated as follows

$$RMSE = \left(\frac{\sum_{i=1}^{N}(x_i - \hat{x}_i)^2}{N}\right)^{0.5} \quad (39)$$

For N the number of SVSF process solving the problem of localization, $\hat{x}_i$ refers to the estimated value, that is the robot pose. Meanwhile x_i is the reference/expected value of the robot pose (when there is no any perturbation). By following this analogy, it clear to have a relationship for all variant value in the input membership function corresponding to output of FIS-IAE ASVSF-based Localization algorithm. The arrangement of DoM in the form of membership function given by GA is presented as follows. Since, there are two different adaption which are relative to R with Q is fixed and relative to Q with R is fixed, therefore, the input arrangement is classified in two types as follows

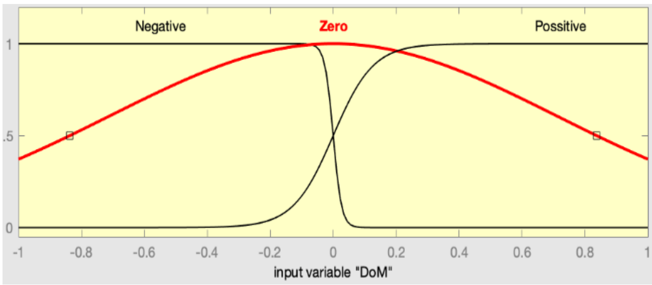

Figure 6 *DoM Arrangement Given By GA-Tuned FIS for Adj relative to R (Q is fixed)*

Fig. 6 shows the result of using GA. The input membership function is adjusted automatically corresponding to the smallest RMSE for estimated robot path.

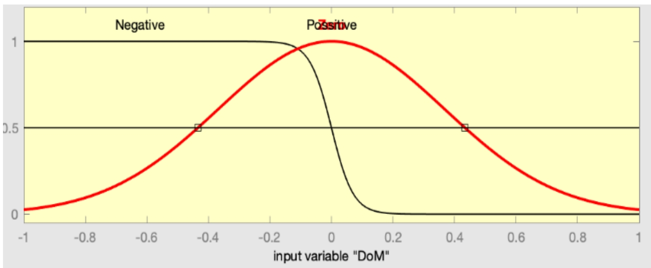

Figure 7 *DoM Arrangement Given By GA-Tuned FIS for Adj relative to Q (R is fixed)*

Fig. 7 depicts the normal arrangement is changed after it is tuned using GA. This membership representation is determined according to the smallest RMSE of the estimated robot path with an adaptation to Q (R is assumed to be fixed). Up to this point, by recalling all principles of localization algorithm in the previous section, the proposed method can be completely depicted in Fig. 8.

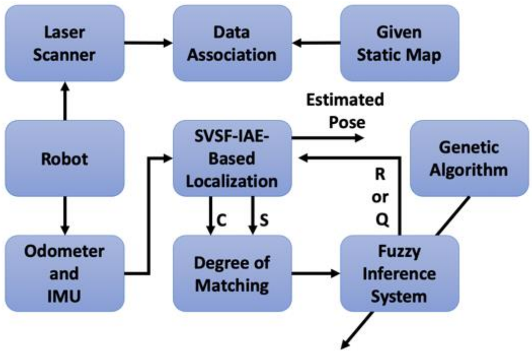

Figure 8 *the Flowchart of Obtaining the Proposed Method*

Results and Discussion

To validate the effectiveness of the proposed method, there are some different performances of analyzed. It includes other conventional method, which are the predecessor of the proposed method, FISGA-IAE ASVSF-Based Localization Algorithm. In detail they are IAE ASVSF, FIS-assisted SVSF-IAE, and FIS-GA-SVSF-IAE. According to characteristic of the IAE, the use of adapted R is considered when Q is fixed, and vise-versa. Therefore, there will be six comparative results. The synthetic simulation is designed for those methods with the following initialization relative to the noise statistic

$$q = [0.3, 0.4]^T, r = \left[0.85, \frac{\pi}{180}\right]^T, Q = \mathrm{diag}(q.^2), R = \mathrm{diag}(.r^2)$$

Secondly, the parameterization for SVSF is sequentially presented as follows

$$\gamma = 0.8, e_{z,0} = [0,0]^T$$

Meanwhile, the locomotion parameters are presented as follows

$$W_r = 30 \text{ cm}$$

Where, it is noted that these parameters are adopted from the real platform dimension and datasheet of Turtlebot2 with the laser scanner is placed on the distance of 5 cm from the body center. The ground truth is given as follows

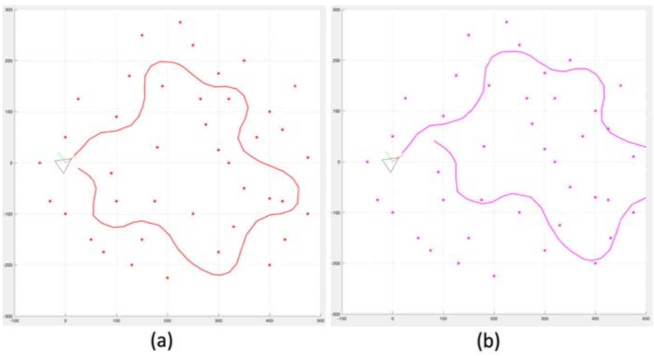

Figure 9 *Ground Truth (a) Expected Path (b) Real Path caused Noisy Process and Measurement*

Fig. 9 depicts that the ground truth in this experiment considered with the known coordinates for all landmarks as the feature-represented map and the robot path is noisy because of some factors. Furthermore, this experiment assumes the robot knows its initial position on the global frame as follows

$$x_{R,0} = \left[\frac{0,0,35\pi}{180}\right]^T, P_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then by performing different algorithms, the comparative result in term of RMSE for robot pose are presented in the following table.

Table 1 *RMSE Values Relative to Estimated Path Based on SVSF-IAE Localization Algorithm*

Localization Algorithm	With Adapted R (Q is fixed)		
	x (cm)	y (cm)	θ (rad)
SVSF-IAE	27.4668	22.3482	0.0016
FIS-assisted SVSF-IAE	20.9244	19.5056	0.0015
FIS-GA-SVSF-IAE	18.1324	18.5161	0.0015

Table 1 shows that the RMSE has been successfully reduced when the IAE-SVSF-based Localization Algorithm is tuned using FIS. It can be seen from the second result in Table result that the RMSE for x-coordinate, y-coordinate, and heading of the robot are smaller than previous one. However, this result should be validated from the perceptive convergence, therefore, the following figure is shown

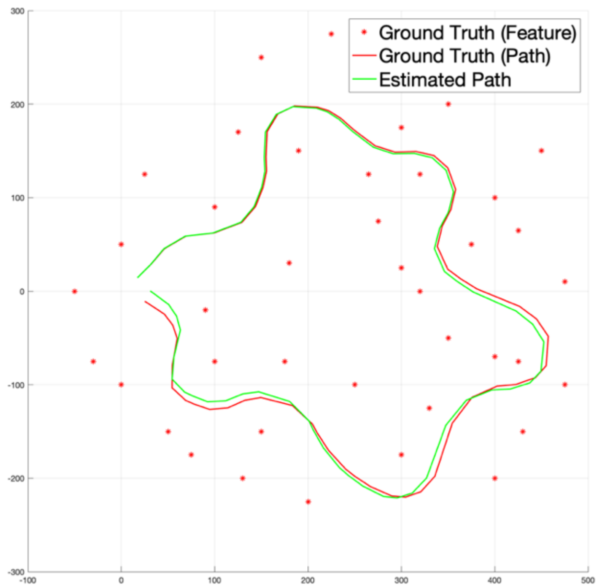

Figure 10 *The Performance of FIS-assisted SVSF-IAE-Based Localization Algorithm with Adapted R*

Fig. 10 shows the performance of FIS-assisted SVSF-IAE-based Localization algorithm. It can be observed from this figure, the estimated values are close to the expected value as presented previously in Fig. 6(a). According to this figure, the performance SVSF-IAE is enhanced with a guaranteed convergence proven by no much deviation from the ground truth. This achievement is obtained based on manual setting of the input membership function, which is believed that it can be more increased when GA is applied. Accordingly, with the same synthetic simulation and initialized parameter, the result of GA-assisted FIS-SVSF-IAE-based Localization algorithm can be seen from the last row in Table 1. It shows that the RMSE relative to the spatial path of the robot are reduced. Again, as an effort to evaluate its effectiveness, the following figure is presented.

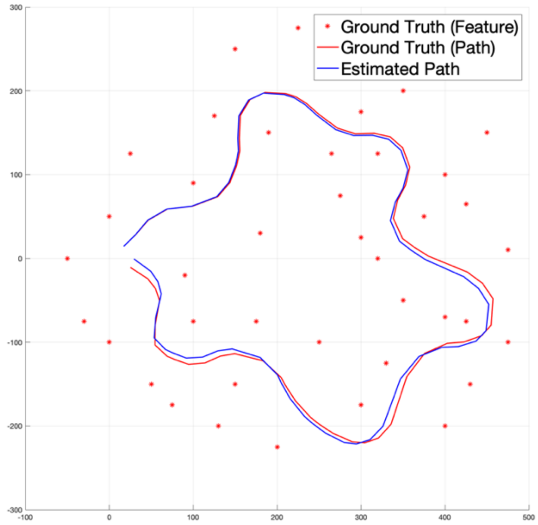

Figure 11 *The Performance of GA-FIS-assisted SVSF-IAE-Based Localization Algorithm with Adapted R*

Fig. 11 depicts the convergence solution offered by the proposed method, which it is not diverge from the ground truth. Up to this point, its accuracy and stability are validated using a term of RMSE when the adapted R is approached. Besides conducting an observation and analysis relative to adaptation R when Q is assumed to be invariant, the role of the proposed method is also analyzed regarding its implementation to recursively update Q with fixed-R. Likely, it is begun by presenting the following table.

Table 2 *RMSE Values Relative to Estimated Path Based on SVSF-IAE Localization Algorithm*

Localization Algorithm	With Adapted Q (R is fixed)		
	x (cm)	y (cm)	θ (rad)
SVSF-IAE	27.4668	22.3482	0.0016
FIS-assisted SVSF-IAE	20.7056	17.8804	0.0013
FIS-GA-SVSF-IAE	17.0180	15.0005	0.0011

Again, the stability of the proposed method is validated according to Table 2. It can be seen from the result in second row that using FIS-assisted SVSF-IAE can sufficiently improve the normal algorithm's performance. Clearly, the small RMSE offered by this algorithm represents the effectiveness of using FIS to the marginalized path both for spatial and heading of robot. It can also be proven from the side-of-view relative to the convergence as shown in the following figure.

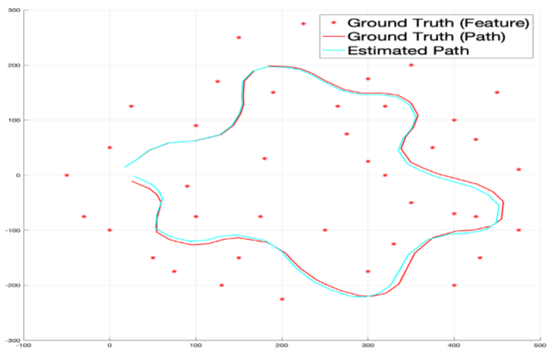

Figure 12 *The Performance of FIS-assisted SVSF-IAE-Based Localization Algorithm with Adapted Q*

Fig. 12 illustrates the stability of FIS-assisted SVSF-IAE-based Localization algorithm in term of RMSE with no divergence detected. However, it can still be improved and tuned using GA once it is done by only using manually setting to DoM's membership function. Similar to the previous analysis, the inference validation can be observed from the RMSE result on third row of Table 2. According to this result, the proposed method is not only showing better performance compared to FIS-assisted SVSF-IAE-based Localization algorithm but also giving the best solution in this experiment, in which it is implicitly stated according to a comparation between R and Q adaptation (see Table 1 and Table 2). This statement is also strengthened with the accuracy of the proposed method in estimating robot path as shown in Fig. 13.

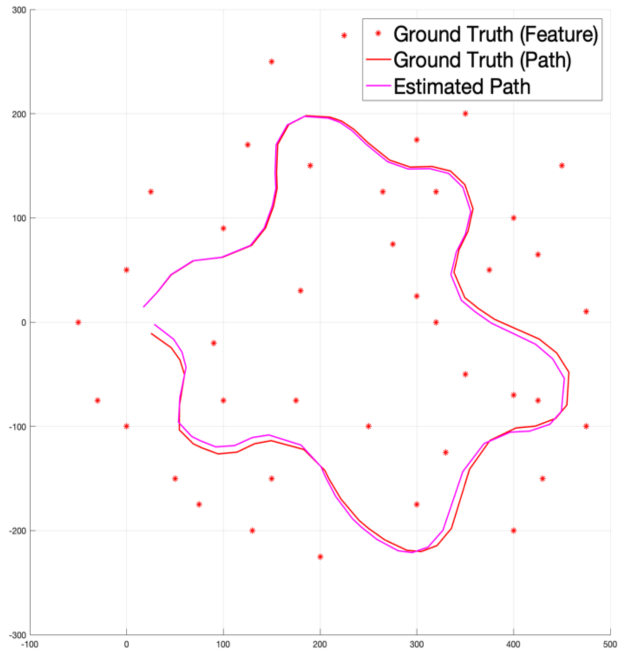

Figure 13 *The Performance of GA-FIS-assisted SVSF-IAE-Based Localization Algorithm with Adapted Q*

Conclusion

This paper presents a novel algorithm for localization based on the enhanced SVSF. Initially, the adaptive formulation of SVSF determined by applying a principle of the Innovation Adaptive Estimation and it is involved as the base. Sequentially, IAE-SVSF is tuned using the Fuzzy Inference System with discrepancies level (defined as DoM) is considered as the input and an adjuster is considered as the output. The discrepancies represent the different between the actual and theoretical covariance of innovation sequence. Meanwhile, the adjuster is determined aiming to rescale the covariance matrix of process Q and measurement R noise statistic. As an effort to increase the accuracy of membership arrangement, the Genetic Algorithm is used to tune the FIS. The idea behind this optimization directly refers to the form of RMSE representing the estimated path of the robot. Furthermore, it is implemented as the core of localization algorithm and compared with other conventional algorithms. Based on the discussion, analysis, and perception of comparatively generated results, the proposed method has been showing a better performance in term of RMSE and convergency which respectively represents the accuracy and stability.

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