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Graph Labelling and Chaldean Numerology in Cryptography

By

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Abstract

Graph labelling, an interesting and developing branch in mathematics has wide range of applications including cryptography. Chaldean numerology, the oldest form of calculation developed by the Babylonians is employed for encoding and decoding messages. In this paper, graph theory with Chaldean numerology and arithmetic number labelling is used for the encryption of text. Python program coding link to decode the message is given.

Keywords: Arithmetic number labelling, Chaldean numerology

Introduction

Graph labelling is assigning of labels represented by integers to vertices[1][2][12] [13],edges[14] [15],faces[3] and blocks[13][14][18][19][21] of a graph. Graph theory finds its applications in many fields like cryptography[4][5][6][9][10][26], medical[17],etc. Cryptography has become the need of the hour since secured transformation of information has become very important. Chaldean numerology [7][8] is an oldest form of calculation used for predicting the future. Arithmetic number labelling was introduced by Uma maheswari and Purnalakshimi in 2022[13]. In this paper, arithmetic number labelling and Chaldean numerology is used for encrypting and decrypting the message.

Definition1: Arithmetic number[1]

A number 'n' is called an Arithmetic number if the arithmetic mean of its divisors is an integer. For example, 6 is an arithmetic number since the arithmetic mean of its divisors 1,2,3,6 is 3, an integer. Some of the arithmetic numbers are 1,3,5,6,7,11,13 .

Definition 2: Arithmetic number labelling [13]

An Arithmetic number labelling of a graph G is a one - to – one function.$f: v(G) \rightarrow W$, where W is the set of whole Numbers.) that induces a bijection $f^*: E(G) \rightarrow (A_1, A_2, A_3, \dots, A_n)$, defined by $f^*(u,v) = |f(u) - f(v)|, \forall\, e = uv \in E(G)$. The graph which admits Arithmetic number labelling is called Arithmetic number graph.

Definition 2: Cartesian product of graph [3]

Let G_1 and G_2 be two simple connected graphs with vertex sets as V_1 and V_2. Then the cartesian product of these two graphs G_1 and G_2 is G_1 x G_2 with vertex set V = V_1 x V_2 and for which two vertices (u_1,u_2) and (v_1,v_2) of G are adjacent if and only if either $u_1 = v_1$ and u_2v_2 belongs to E (G_2), or $u_2 = v_2$ and u_1v_1 belongs to E(G_1) .

Definition 3[20]: Ciphertext is the required version of plain text.
Definition 4[20]: Encryption is the process of transforming plain text into ciphertext.
Definition 5[20]: Decryption is the process of transforming cipher text into plain text.
Definition 6[20]: Key is the most essential tool which encodes the plain text and decodes the ciphertext.

Chaldean numerology

Table 1

A	B	C	D	E	F	G	H	I	J
1	2	3	4	5	8	3	5	1	1
K	L	M	N	O	P	Q	R	S	T
2	3	4	5	7	8	1	2	3	4
U	V	W	X	Y	Z				
6	6	6	5	1	7				

Main results

In this section, Arithmetic number labelling and Chaldean numbers are used for encryption of cipher text. New algorithm is developed to secure transmission of codes. The following is the list of instructions involved in the process of encryption.

The Chaldean numbers associated with alphabets are shown in table 2.

Table 2

1	2	3	4	5	6	7	8
A, I, J, Q, Y	B, K, R	C, G, L, S	D, M, T	E, H, N, X	U, V, W	O, Z	F, P

To enable uniqueness in vertex labelling, the numbers are associated with the alphabets in the following manner shown in table 3.

Table 3

1	2	3	4	5	6	7	8
A -11	B -21	C - 31	D – 41	E -51	U -61	O -71	F -81
I -12	K -22	G - 32	M – 42	H -52	V -62	Z -72	P -82
J -13	R – 23	L - 33	T – 43	N -53	W -63		
Q -14		S - 34		X -54			
Y -15							

Encryption algorithm

1. The text message intended for the receiver is sent as cartesian product of 2 graphs. One is a path graph with arithmetic number labelling, and another is a closed graph with Chaldean numbers associated with the alphabets of the text message. The text message should contain alphabets which do not repeat.
2. This graph is converted into a 5 x 5 matrix if the text has 5 letters. Name this matrix as A
3. The key of encoding is a 5 x 5 matrix named B. Here we use the upper triangular matrix with even numbers. B as

$$\begin{pmatrix} 2 & 4 & 6 & 8 & 10 \\ 0 & 12 & 14 & 16 & 18 \\ 0 & 0 & 20 & 22 & 24 \\ 0 & 0 & 0 & 26 & 28 \\ 0 & 0 & 0 & 0 & 30 \end{pmatrix}$$

4. M = AB which is a 5 x 5 matrix.
5. The elements of this 5 x 5 matrix is assigned as vertices to the cartesian product of graphs G_1 x G_2
6. The graph G_1 x G_2 is the encrypted message.

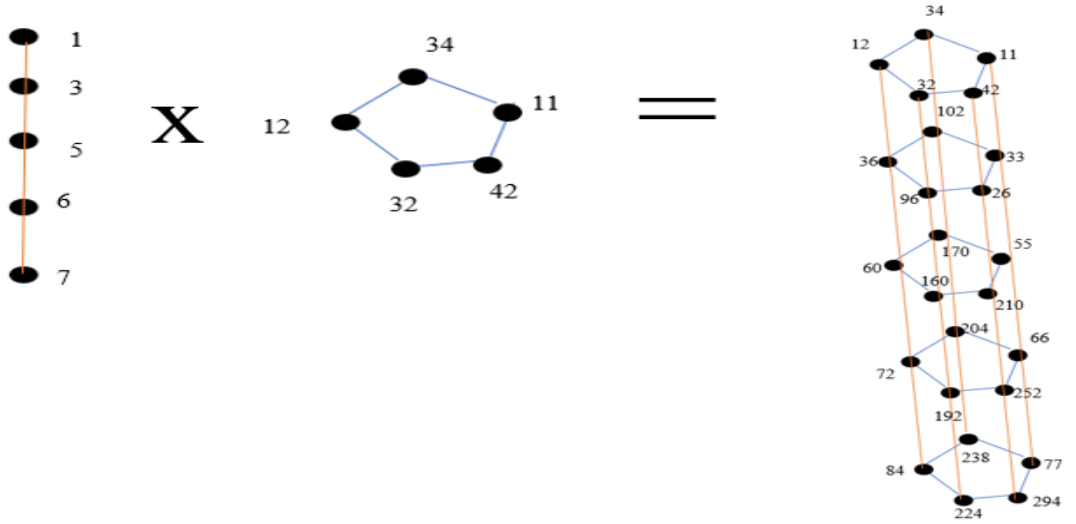
Decryption algorithm

1. The vertices of the received graph H_1 x H_2 is converted into a 5 x 5 matrix with the vertices as elements of a matrix.
2. This is matrix M.
3. This matrix M is multiplied with the inverse of Key matrix B (i.e) $A = MB^{-1}$
4. Once we get the matrix A ,we do the following
5. $CN_{11} = a_{11}/ A_1$, $CN_{12} = a_{12}/ A_2$, $CN_{13} = a_{13}/ A_3$, $CN_{14} = a_{14}/ A_4$, $CN_{15} = a_{15}/ A_5$.
6. Note CN_{11} denotes the alphabet used in table 3 associated with the Chaldean numerology.

ustration

Let us assume that the message sent to the receiver is SIGMA
SIGMA
The numbers associated with the alphabets from table 3 is as follows
S – 34, I -12, G -32, M – 42, A -11.

We co



= nstruct a graph G_1 x G_2

$$\begin{bmatrix} 34 & 12 & 32 & 42 & 11 \\ 102 & 36 & 96 & 26 & 33 \\ 170 & 60 & 160 & 210 & 55 \\ 204 & 72 & 192 & 252 & 66 \\ 238 & 84 & 224 & 294 & 77 \end{bmatrix}$$

Vertices of this graph is converted to a matrix. A =

$$\begin{bmatrix} 2 & 4 & 6 & 8 & 10 \\ 0 & 12 & 14 & 16 & 18 \\ 0 & 0 & 20 & 22 & 24 \\ 0 & 0 & 0 & 26 & 28 \\ 0 & 0 & 0 & 0 & 30 \end{bmatrix}$$

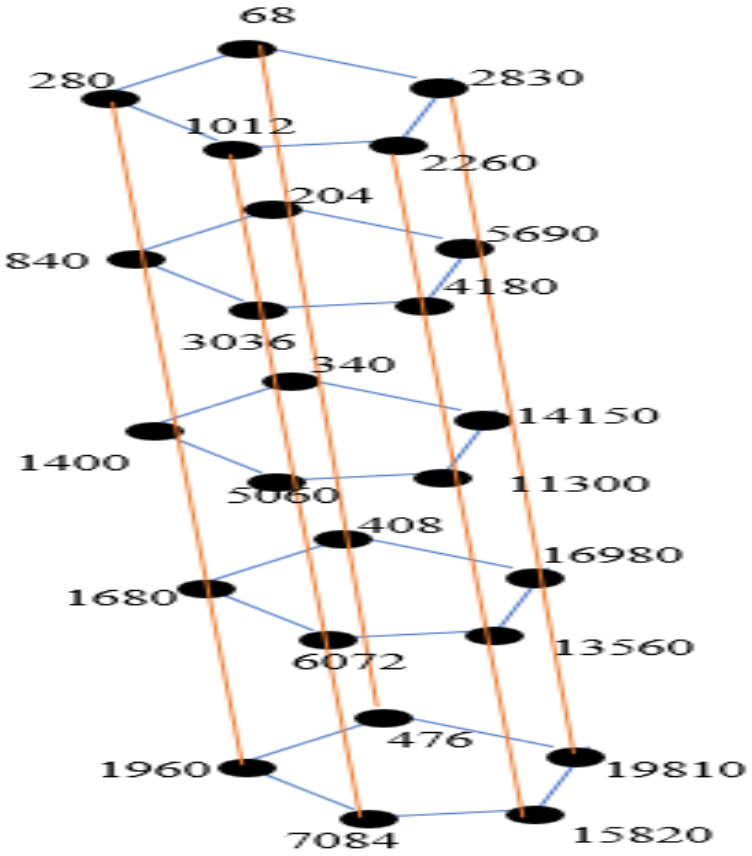
Consider the key for encoding matrix B =

$$\begin{bmatrix} 34 & 12 & 32 & 42 & 11 \\ 102 & 36 & 96 & 26 & 33 \\ 170 & 60 & 160 & 210 & 55 \\ 204 & 72 & 192 & 252 & 66 \\ 238 & 84 & 224 & 294 & 77 \end{bmatrix} \times \begin{bmatrix} 2 & 4 & 6 & 8 & 10 \\ 0 & 12 & 14 & 16 & 18 \\ 0 & 0 & 20 & 22 & 24 \\ 0 & 0 & 0 & 26 & 28 \\ 0 & 0 & 0 & 0 & 30 \end{bmatrix}$$

Compute M = AB

$$\begin{bmatrix} 68 & 280 & 1012 & 2260 & 2830 \\ 204 & 840 & 3036 & 4180 & 5690 \\ 340 & 1400 & 5060 & 11300 & 14150 \\ 408 & 1680 & 6072 & 13560 & 16980 \\ 476 & 1960 & 7084 & 15820 & 19810 \end{bmatrix}$$

The matrix M when converted to a graph is



M =

Now we will decode the graph

The vertices of the graph are converted to a matrix.

$$\begin{bmatrix} 68 & 280 & 1012 & 2260 & 2830 \\ 204 & 840 & 3036 & 4180 & 5690 \\ 340 & 1400 & 5060 & 11300 & 14150 \\ 408 & 1680 & 6072 & 13560 & 16980 \\ 476 & 1960 & 7084 & 15820 & 19810 \end{bmatrix}$$

M =

Since M =A B^{-1} we multiply M with B^{-1} which is

$$\begin{bmatrix} 1/2 & -1/6 & -\frac{1}{30} & -\frac{3}{30} & -\frac{6}{325} \\ 0 & \frac{1}{12} & -\frac{7}{120} & -\frac{1}{520} & -\frac{1}{650} \\ 0 & 0 & 1/20 & \frac{-11}{260} & -\frac{1}{1950} \\ 0 & 0 & 0 & \frac{1}{26} & -\frac{7}{195} \\ 0 & 0 & 0 & 0 & \frac{1}{30} \end{bmatrix}$$

Hence, we get

$$\begin{bmatrix} 68 & 280 & 1012 & 2260 & 2830 \\ 204 & 840 & 3036 & 4180 & 5690 \\ 340 & 1400 & 5060 & 11300 & 14150 \\ 408 & 1680 & 6072 & 13560 & 16980 \\ 476 & 1960 & 7084 & 15820 & 19810 \end{bmatrix} \text{ x } \begin{bmatrix} 1/2 & -1/6 & -\frac{1}{30} & -\frac{3}{30} & -\frac{6}{325} \\ 0 & \frac{1}{12} & -\frac{7}{120} & -\frac{1}{520} & -\frac{1}{650} \\ 0 & 0 & 1/20 & \frac{-11}{260} & -\frac{1}{1950} \\ 0 & 0 & 0 & \frac{1}{26} & -\frac{7}{195} \\ 0 & 0 & 0 & 0 & \frac{1}{30} \end{bmatrix}$$

$$\begin{bmatrix} 34 & 12 & 32 & 42 & 11 \\ 102 & 36 & 96 & 26 & 33 \\ 170 & 60 & 160 & 210 & 55 \\ 204 & 72 & 192 & 252 & 66 \\ 238 & 84 & 224 & 294 & 77 \end{bmatrix}$$

So, we got the original matrix A. We can use any row to decode. We now consider the elements in the first row which are 34,12,32,42,11.

Alphabets associated to this Chaldean numerology (CN) is as follows:

$CN_1 = \frac{34}{A_1} = \frac{34}{1} =$ **34** which is associated to S from Table 3

Similarly

$CN_2 = \frac{12}{1} = \frac{12}{1} =$ **12** which is associated to I from Table 3

$CN_3 = \frac{32}{1} = \frac{32}{1} =$ **32** which is associated to G from Table 3

$CN_4 = \frac{42}{11} = \frac{42}{11} =$ **42** which is associated to M from Table 3.

$CN_5 = \frac{11}{A_1} = \frac{11}{1} =$ **11** which is associated to A from Table 3.

We use the elements in the third row to decode.

$CN_1 = \frac{170}{A_3} = \frac{170}{5} =$ **34** which is associated to S from Table 3.

$CN_2 = \frac{60}{A_3} = \frac{60}{5} =$ **12** which is associated to I from Table 3.

$CN_3 = \frac{160}{A_3} = \frac{160}{5} =$ **32** which is associated to G from Table 3.

$CN_4 = \frac{210}{A_3} = \frac{210}{5} =$ **42** which is associated to M from Table 3.

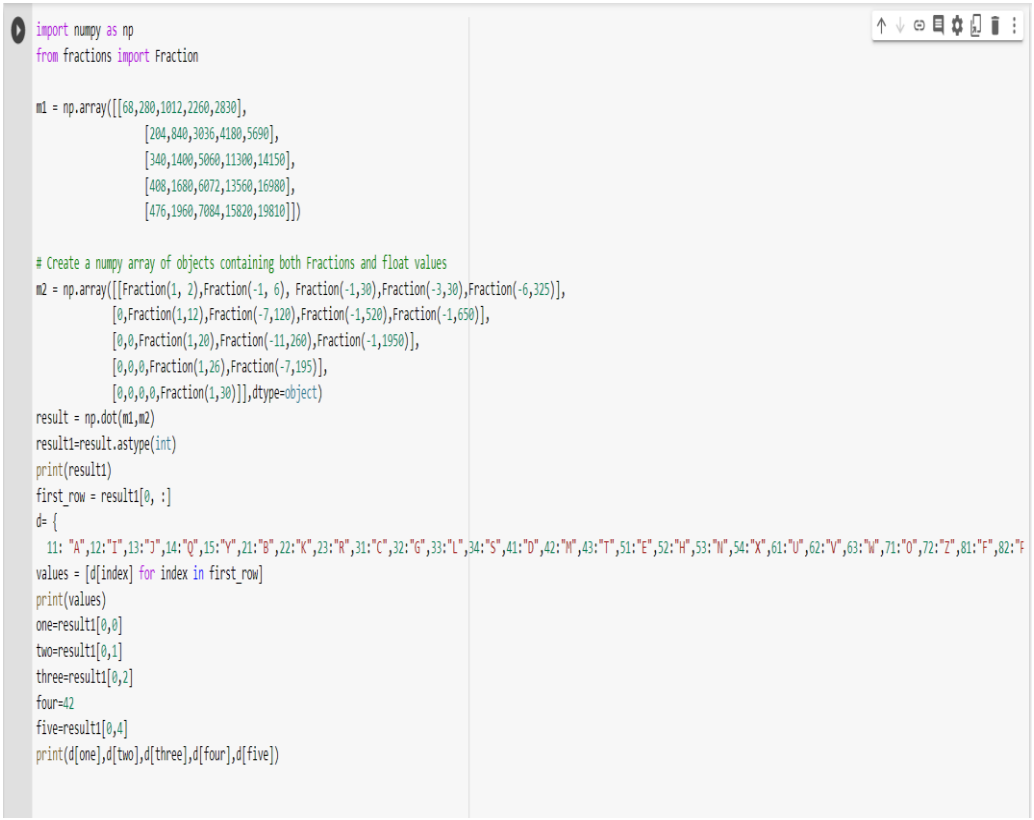
$CN_5 = \frac{55}{A_3} = \frac{55}{5} =$ **11** which is associated to S from Table 3.

Likewise, we can use any row for decoding. For convenience we can use the row which has smaller numbers.

Hence, we have encrypted and decrypted the word ‘SIGMA’. To simplify the process of decoding, we can use Python program. Python program coding link to decode a five letter message is given below.

https://colab.research.google.com/drive/1KVZomOYqL8hbWkW70ITN5ZFxdkyPCCeR?usp=sharing

screenshot of the program.



```
import numpy as np
from fractions import Fraction

m1 = np.array([[68,280,1012,2260,2830],
               [204,840,3036,4180,5690],
               [340,1400,5060,11300,14150],
               [408,1680,6072,13560,16980],
               [476,1960,7084,15820,19810]])

# Create a numpy array of objects containing both Fractions and float values
m2 = np.array([[Fraction(1, 2),Fraction(-1, 6), Fraction(-1,30),Fraction(-3,30),Fraction(-6,325)],
            [0,Fraction(1,12),Fraction(-7,120),Fraction(-1,520),Fraction(-1,650)],
            [0,0,Fraction(1,20),Fraction(-11,260),Fraction(-1,1950)],
            [0,0,0,Fraction(1,26),Fraction(-7,195)],
            [0,0,0,0,Fraction(1,30)]],dtype=object)
result = np.dot(m1,m2)
result1=result.astype(int)
print(result1)
first_row = result1[0, :]
d= {
  11: "A",12:"I",13:"J",14:"Q",15:"Y",21:"B",22:"K",23:"R",31:"C",32:"G",33:"L",34:"S",41:"D",42:"M",43:"T",51:"E",52:"H",53:"N",54:"X",61:"U",62:"V",63:"W",71:"O",72:"Z",81:"F",82:"F
values = [d[index] for index in first_row]
print(values)
one=result1[0,0]
two=result1[0,1]
three=result1[0,2]
four=42
five=result1[0,4]
print(d[one],d[two],d[three],d[four],d[five])
```

Conclusion

In this paper, a new algorithm for coding and decoding is introduced. The algorithm involves arithmetic number labeling and Chaldean numerology. This algorithm for encoding and decoding involving graph labeling is more secure and simple. In this work coding using Python is employed to simplify the process of decoding. There is scope for creating more algorithms using graph operations for encryption and decryption.

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