A Production inventory model for deteriorating items with quadratic time dependent demand with two levels of Production

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Abstract.In many production systems, it is not always feasible to maintain a single rate of production during the entire production period. The production of items at various rates during sub-periods to meet various constraints arising from changes in demand patterns and market fluctuations. Therefore, in this paper, we developed a two rates of production inventory model with consistent deterioration rate. The demand rate is assumed to be quadratic function of time and shortages are not allowed. This model took into accounts circumstances when production began at one rate and then changed to another rate after a specific amount of time. Such a scenario is preferable in that it avoids building up a large stock of manufacturing items at initial stage which reduce holding cost. The goal of the study is to determine optimal solution of cycle run length which minimizes manufacturer's total cost. To verify the result a numerical and sensitivity analysis are provided.

Keywords:Production, Inventory, Deterioration, Quadratic Demand and Cost.

1. Introduction

Inventories are items that are kept for potential use or sale. Businesses require effective inventory management to control inventory costs and preserve the continuity of their operations. Purchase, order, hold, and shortfall charges are the main components of inventory expenses.Inventory management refers to the process of overseeing and controlling the flow of products and material within a business or organization. Goyal [1] developed a mathematical model to calculate the economic order quantity for a product when the supplier allows a fixed delay in payment. Aggarwal and Jaggi [2] later expanded Goyal's model to include deteriorative items. Chen J [3] presents a comprehensive dynamic programming model for managing inventory items subject to Weibull-distributed deterioration. Mariappan et al.[4]developed a lot-sizing model for deteriorating items by allowing the demand and the deterioration time to be stochastic and follows an exponential distribution.Khanna and Chaudhuri[5] addresses an inventory problem at the order level, where the demand rate is described by a continuous quadratic function of time. Samanta and Roy [6] proposed an inventory model for a two-level production system that addressed deteriorating items and shortage. Kalam et.al. [7] consider the production inventory problem in which the deterioration is weibull distribution, production and demand are quadratic function of time.Tripathy and Mishra [8]considered two cases; those are for the case payment within the permissible time and for payment after the expiry of permissible time. Subsequently, Bhowmic etal. [9] extended the model proposed by Samanta and Roy to incorporate a variable production cycle. Amutha& Chandrasekaran [10] used exponential distribution in their model to represent the distribution of time to deterioration.A deterministic inventory model is investigated for deteriorating items in which the demand is time quadratic and shortages are allowed and partially backlogged by Jagdeeswari et. al. [11]. An inventory model developed for deteriorating items that deteriorates at a Weibull distributed rate, assuming the demand rate as a ramp type function of time by Sharma et al. [12]. Sivashankari and Panayappan [13] worked on a research paper, in that research paper a production inventory model with deteriorative items in which two different rates of productions are considered. Arif [14] meticulously investigated the degradation rate of the form within the context of the three-parameter Weibull distribution, as expounded in their scholarly article. Furthermore, their research comprehensively addressed the explicit consideration of transportation costs in the context of replenishing the order quantity. Kumar et al. [15] presented a production inventory model that was developed by considering two different rates of production with exponential demand rate. Singh [16] focused on a production-inventory model in which demand is contingent upon both the stock level and the selling price of the item. Specifically, the demand rate exhibits a linear increase concerning stock and time, while it decreases concerning the selling price of the item.Meena et al. [17] illustrate a non-instantly degradable product inventory system is built with a price-sensitive demand and a Weibull credit term allocation reduction rate. Some backlogged deficiencies are permitted.Maity et al. [18] explored an inventory model for economic order quantity, specifically focusing on imperfect items in the context of non-random and uncertain demand. In this scenario, customers play a role in screening imperfect items during the selling period. Maity et al. [19] introduced the concept of a parabolic dense fuzzy set and examines its fundamental arithmetic operations through graphical representations. Additionally, the article integrates the notion of a lock set within the framework of a parabolic dense fuzzy set. Rahaman et al. [20]worked on an economic production quantity (EPQ) model incorporating the element of deterioration. This model features a production rate that varies with the current stock level and a demand rate contingent upon both the unit selling price and the stock level.Singh et al. [21] examined a production inventory model with three different levels of deterioration rates. This model plays a pivotal role in the manufacturing process of boards and assembly units.Singh et al. [22] studied a mathematical inventory model where the demand is considered a function of the selling price, indicating its dependence on the selling price. Also, the holding cost is assumed to be a linear function of time. Maity et al. [23] developed an environmentally conscious inventory model, wherein the demand rate is contingent upon factors such as the selling price, current stock levels, and the level of environmental consciousness.

The demand for any given article within market or industrial settings is inherently dynamic, exhibiting variability across distinct time intervals. To capture this inherent variability, we employ a quadratic form to model demand as a function dependent on time. In the framework of this study, the demand rate is postulated to manifest as a quadratic function of time, with a strict prohibition on shortages. This model incorporates the intricate dynamics of variable production rates over different time intervals, effectively implementing a two-tiered production approach. This scenario is favored for its ability to avert the accumulation of a substantial inventory of manufactured goods during the initial phase, thereby mitigating associated holding costs

This research article is structured into the following eight sections.: -

Notations and Assumptions used for calculation in systematic way displayed inSection 2.

The mathematical formulation of our model, along with various types of costs, isderived in section 3. Additionally, the solution is obtained. In section 4 Two numerical examples with specific values are employed to validate theproposed model.The sensitivity of our problem is demonstrated section 5 through a table created by varyingthe values of parameters used in this model. The section 6 presents graphs of the minimized cost function concerning variousparameters, along with their corresponding observations. Additionally, straightforward graphs for sensitivity analysis are included. Next in the section 7 a visual conclusion is provided. Section 8 covered novelty of article and Section 9 covered conclusion of our model, with relevance to its future importance.

2.Assumption and Notations

This inventory model has considered the following assumptions and notations.

2.1 Assumptions

The assumption of this inventory models is as follow: -

1. The demand rate is known, quadratic and continuous.
2. Two rates of production are considered.
3. Rate of production is known and constant.
4. The replenishment occurs instantaneous i.e., replenishment rate is infinite.
5. The production rate is always greater the demand rate.
6. The items is single product; it does not interact with any other inventory items
7. Lead time is assumed to be zero.
8. It is assumed that no repair or replacement of the deteriorative items t during take place during a given cycle.
9. Holding cost is constant.

2.2 Notations

(1) Sc –Ordering Cost / Set up Cost per unit per order.

(2) κ - Production Rate (units per unit time) during time interval $[0, t_1]$
$a\kappa$ - Production Rate (units per unit time)during time interval $[t_1, t_2]$, where $a > 0$
(3) C_P - Purchase Cost per unit.
(4) D_C– Cost of deteriorated items.
(5) λ- The Rate of deterioration, where $0 <\lambda< 1$
(6) H_C= Unit Holding Cost per unit per unit of time.
(7) t_3 - Production Cycle length (Total Cycle time)
(8) D(t) - The demand rate, $D(t) = c_1 + c_2t + c_3t^2$ where c_1, c_2 and $c_3 > 0$
c_1=initial demand rate (units per unit time), c_2=initial rate of change in demand., c_3=accretion of demand rate.
(9) I(t) - Inventory level at time t.
(10) $I_1(t)$ - Inventory level at time t during time interval $[0, t_1]$
(11) $I_2(t)$ - Inventory level at time t during time interval $[t_1, t_2]$
(12) $I_3(t)$ - Inventory level at time t during time interval $[t_2, t_3]$
(13) t_3- Time at which inventory level reaches to zero.
(14) IM_1-Maximum inventory level at $[0, t_1]$
(15) IM_2 -Maximum inventory level at $[t_1, t_2]$
(16) Q* - Order quantity during the cycle length t_3
(17) TAC^* - Total Cost / cycle time
(18) w.r.t. – with respect to

Leveraging the assumptions and notations, we endeavor to formulate a mathematical inventory model aimed at attaining an optimal solution for the Economic Production Quantity (EPQ).

3 Mathematical formulations of the proposed Model

In our model, the initial inventory level is 0 at time t=0. After a time at time $t=t_1$ the inventory level reaches I_{M1}. After some time at time $t=t_2$ the inventory level is I_{M2}. Subsequently, with the impact of deterioration and demand the inventory level reaches to zero at time $t=t_3$

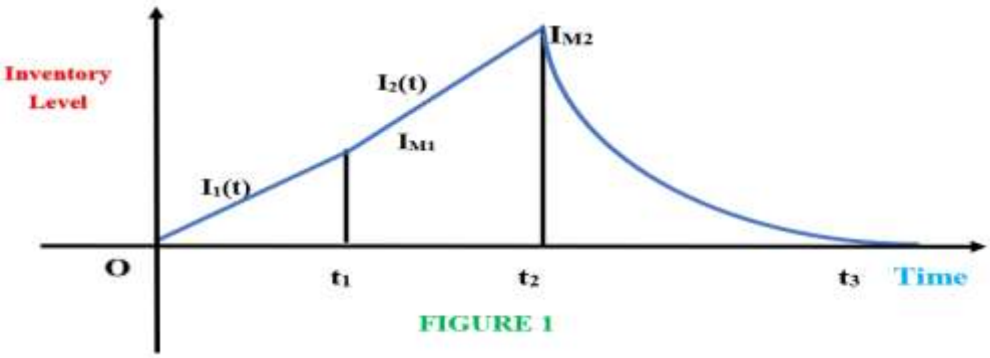


Figure 1: Proposed inventory model with Two Level of Production

3.1 Mathematical Model

The differential equation during the time interval $[0,t_1]$ is presented as follows: -

$$\frac{dI_1(t)}{dt}+\lambda I_1(t) = \kappa - D(t) = \kappa-\left(c_1+c_2t+c_3t^2\right) \quad (1)$$

The differential equation during the time interval $[t_1, t_2]$is presented as follows: -

$$\frac{dI_2(t)}{dt}+\lambda I_2(t) = a\kappa - D(t) = a\kappa-\left(c_1+c_2t+c_3t^2\right) \quad (2)$$

The differential equation during the time interval $[t_2, t_3]$is presented as follows: -

$$\frac{dI_3(t)}{dt}+\lambda I_3(t) = -D(t) = -\left(c_1+c_2t+c_3t^2\right) \quad (3)$$

With Boundary Conditions the interval $[0, T_1]$

$$I_1(t)=0 \quad at\ t=0 \quad and\ I_1(t)=I_{M1} \quad at \quad t=t_1 \quad (4)$$

And Boundary Conditions in the interval $[t_1, t_2]$

$$I_2(t)=I_{M2} \quad at \quad t=t_2 \quad (5)$$

And Boundary Conditions in the interval $[t_2, t_3]$

$$I_3(t)=0 \quad at\ t=t_3 \quad and\ I_3(t)=I_{M2} \quad at \quad t=t_2 \quad (6)$$

Solution of equation (1) with help of boundary conditions given in (4), we get

$$I_1(t)=\frac{1}{\lambda^3}\left\{\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)+2c_3\lambda t-c_2\lambda^2t-c_3\lambda^2t^2-e^{-\lambda t}\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)\right\} \quad (7)$$

$$I_{M1}=\frac{1}{\lambda^3}\left\{\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)+2c_3\lambda t_1-c_2\lambda^2t_1-c_3\lambda^2t_1^{\,2}-e^{-\lambda t_1}\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)\right\} \quad (8)$$

Result number (7) and (8) are the solutions of equation number (1)

Solution of equation (2) with help of boundary conditions given in (5), and use of equation (8) we get

$$I_2(t)=\frac{1}{\lambda^3}\Big[\left\{\left(a\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)+2c_3\lambda t-c_2\lambda^2t-c_3\lambda^2t^2\right\} \\ +\kappa\lambda^2(1-a)e^{\lambda(t_1-t)}-e^{-\lambda(2t_1-t)}\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)\Big] \quad (9)$$

And we find

$$I_{M2}=\frac{1}{\lambda^3}\Big[\Big\{\left(a\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)+2c_3\lambda t_2-c_2\lambda^2 t_2-c_3\lambda^2 t_2^2\Big\}$$
$$+\kappa\lambda^2(1-a)e^{\lambda(t_1-t_2)}-e^{-\lambda(2t_1-t_2)}\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)\Big]$$

(10)

Result number (9) and (10) are the solutions of equation number (2)

Solution of equation (3) with help of boundary conditions given in (6), we get

$$I_3(t)=\frac{1}{\lambda^3}\begin{bmatrix}\left(-c_1\lambda^2+c_2\lambda-2c_3\right)+2c_3\lambda t-c_2\lambda^2 t-c_3\lambda^2 t^2-e^{\lambda(t_3-t)} \\ \Big\{\left(-c_1\lambda^2+c_2\lambda-2c_3\right)+2c_3\lambda t_3-c_2\lambda^2 t_3-c_3\lambda^2 t_3^{\,2}\Big\}\end{bmatrix} \quad (11)$$

Result number (11) is the solutions of equation number (3)

3.3 Cost Calculation of Proposed Model

To calculation of different types of costs, of the proposed model which arefollows:

(i) **Ordering Cost / Set Up Cost (O.C.)** = S_c

(iii) **Purchase Cost (P.C.)** = $C_P.\ Q^*$

$$=\frac{C_P}{\lambda^3}\Big[\Big\{\left(a\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)+2c_3\lambda t_2-c_2\lambda^2 t_2-c_3\lambda^2 t_2^2\Big\}$$
$$+\kappa\lambda^2(1-a)e^{\lambda(t_1-t_2)}-e^{-\lambda(2t_1-t_2)}\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)\Big] \quad (12)$$

(ii)**The order size during the period [0, t_3] = Q^***= I_{M2}

$$Q^*=\frac{1}{\lambda^3}\Big[\Big\{\left(a\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)+2c_3\lambda t_2-c_2\lambda^2 t_2-c_3\lambda^2 t_2^2\Big\}$$
$$+\kappa\lambda^2(1-a)e^{\lambda(t_1-t_2)}-e^{-\lambda(2t_1-t_2)}\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)\Big] \quad (13)$$

(iv) **The Holding Cost (H.C.)** = $\int_0^{t_3} \mathrm{H}_C\,.\mathrm{I}(t)dt$

$$=\int_0^{t_1}\mathrm{H}_C\,.\mathrm{I}_1(t).dt+\int_{t_1}^{t_2}\mathrm{H}_C\,.\mathrm{I}_2(t).dt+\int_{t_2}^{t_3}\mathrm{H}_C\,.\mathrm{I}_3(t).dt$$

$$=\frac{H_C}{\lambda^3}\left[\begin{array}{l}\left\{\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)t_1+\left(2c_3\lambda-c_2\lambda^2\right)\left(\frac{t_1^2}{2}\right)+\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)\right.\\ \left.\left(\frac{e^{-\lambda t_1}-1}{\lambda}\right)-\frac{c_3\lambda^2 t_1^3}{3}\right\}+\left\{\left(a\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)\left(t_2-t_1\right)+\left(2c_3\lambda-c_2\lambda^2\right)\right.\\ \left(\frac{t_2^2-t_1^2}{2}\right)-\frac{c_3\lambda^2}{3}\left(t_2^3-t_1^3\right)+\kappa\lambda^2\left(a-1\right).\left(e^{\lambda(t_1-t_2)}-1\right)+\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)\\ \left.\left(\frac{e^{-\lambda(2t_1-t_2)}-e^{-\lambda t_1}}{\lambda}\right)\right\}+\left\{\left(-c_1\lambda^2+c_2\lambda-2c_3\right)\left(t_3-t_2\right)+\left(2c_3\lambda-c_2\lambda^2\right)\left(\frac{t_3^2-t_2^2}{2}\right)-\right.\\ \left.\frac{c_3\lambda^2}{3}\left(t_3^3-t_2^3\right)+\left(-c_1\lambda^2+c_2\lambda-2c_3+2c_3\lambda t_3-c_2\lambda^2 t_3-c_3\lambda^2 t_3^2\right)\left(\frac{1-e^{\lambda(t_3-t_2)}}{\lambda}\right)\right\}\end{array}\right]$$

(15)

(v) **The deterioration cost for the inventory during [0, t₃]** **D.C.** $= D_C.\lambda.\int_0^{t_3} \mathrm{I}(t)dt$

$$=\frac{D_C}{\lambda^2}\left[\begin{array}{l}\left\{\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)t_1+\left(2c_3\lambda-c_2\lambda^2\right)\left(\frac{t_1^2}{2}\right)+\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)\right.\\ \left.\left(\frac{e^{-\lambda t_1}-1}{\lambda}\right)-\frac{c_3\lambda^2 t_1^3}{3}\right\}+\left\{\left(a\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)\left(t_2-t_1\right)+\left(2c_3\lambda-c_2\lambda^2\right)\right.\\ \left(\frac{t_2^2-t_1^2}{2}\right)-\frac{c_3\lambda^2}{3}\left(t_2^3-t_1^3\right)+\kappa\lambda^2\left(a-1\right).\left(e^{\lambda(t_1-t_2)}-1\right)+\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)\\ \left.\left(\frac{e^{-\lambda(2t_1-t_2)}-e^{-\lambda t_1}}{\lambda}\right)\right\}+\left\{\left(-c_1\lambda^2+c_2\lambda-2c_3\right)\left(t_3-t_2\right)+\left(2c_3\lambda-c_2\lambda^2\right)\left(\frac{t_3^2-t_2^2}{2}\right)-\right.\\ \left.\frac{c_3\lambda^2}{3}\left(t_3^3-t_2^3\right)+\left(-c_1\lambda^2+c_2\lambda-2c_3+2c_3\lambda t_3-c_2\lambda^2 t_3-c_3\lambda^2 t_3^2\right)\left(\frac{1-e^{\lambda(t_3-t_2)}}{\lambda}\right)\right\}\end{array}\right] \quad (16)$$

(vi) **Total Average Inventory Cost (T.A.C.)**

$=\frac{1}{t_3}$[O.C.+ H.C.+D.C.+P.C.] $=\frac{1}{t_3}$ (Total Cost) (17)

$$T.C.=\left[\begin{array}{l}\left(\frac{H_C+\lambda D_C}{\lambda^3}\right)\left[\left\{\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)t_1+\left(2c_3\lambda-c_2\lambda^2\right)\left(\frac{t_1^2}{2}\right)+\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-\right.\right.\right.\\ \left.\left.2c_3\right)\left(\frac{e^{-\lambda t_1}-1}{\lambda}\right)-\frac{c_3\lambda^2t_1^3}{3}\right\}+\left\{\left(a\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)\left(t_2-t_1\right)+\left(2c_3\lambda-c_2\lambda^2\right)\left(\frac{t_2^2-t_1^2}{2}\right)\right.\\ \left.-\frac{c_3\lambda^2}{3}\left(t_2^3-t_1^3\right)+\kappa\lambda^2\left(a-1\right).\left(e^{\lambda(t_1-t_2)}-1\right)+\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)\left(\frac{e^{-\lambda(2t_1-t_2)}-e^{-\lambda t_1}}{\lambda}\right)\right\}\\ +\left\{\left(-c_1\lambda^2+c_2\lambda-2c_3\right)\left(t_3-t_2\right)+\left(2c_3\lambda-c_2\lambda^2\right)\left(\frac{t_3^2-t_2^2}{2}\right)-\frac{c_3\lambda^2}{3}\left(t_3^3-t_2^3\right)+\left(-c_1\lambda^2+c_2\lambda\right.\right.\\ \left.\left.\left.-2c_3+2c_3\lambda t_3-c_2\lambda^2t_3-c_3\lambda^2t_3^2\right)\left(\frac{1-e^{\lambda(t_3-t_2)}}{\lambda}\right)\right\}\right]+\frac{C_P}{\lambda^3}\left[\left\{\left(a\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)+2c_3\lambda t_2\right.\right.\\ \left.\left.-c_2\lambda^2t_2-c_3\lambda^2t_2^2\right\}+\kappa\lambda^2(1-a)e^{\lambda(t_1-t_2)}-e^{-\lambda(2t_1-t_2)}\left(\kappa\lambda^2-c_1\lambda^2+c_2\lambda-2c_3\right)\right]+S_C\end{array}\right] \quad (18)$$

4. Solution Algorithm: -

The solution algorithm of our proposed model is given below.

Step 1. Calculate different type of Costs using Equation (12) to (17).

Step 2. Calculate Total average inventory using equation (18).

Step 3. Find first derivative of TAC w.r.t. t_3. And after find first derivative calculate positive value of t_3. We find the value of t_3 is 8.16

Step 4. Calculate $\frac{d^2(T.A.C.)}{dt_3^2}$ and we find that $\frac{d^2(T.A.C.)}{dt_3^2}>0$ at the point $t_3=8.16$

Step 5. Employing the mathematical software MATLAB (R2021a) and putting the value of α= 0.90, c_1 = 50, c_2= 30, c_3 =40, κ=500 per unit time, t_1=1, t_2=5, λ=0.95, C_p=20 per unit, S_c=100 per unit, H_C=100 per unit time, D_C=50 per unit in equation of TAC.

Step 6.and after solving we find the value of TAC is 5789.6

Step 7. Change values of different parameters with rate of +20%, +10%, -10%, -20% and find different results. From these results we construct sensitivity analysis.

Step 8. We get desired results.

4.1 Numerical Example number 1

Should we consider a numerical illustration and employ select parameter values inherent to our inventory model: -

α= 0.90, c_1 = 50, c_2= 30, c_3 =40, κ=500 per unit time, t_1=1, t_2=5, λ=0.95, C_p=20 per unit, S_c=100 per unit, H_C=100 per unit time, D_C=50 per unit.
Subsequently, we insert these specific values into equations numbered (15) and (18). Employing the mathematical software MATLAB (R2021a), we tackle this problem, yielding the following optimal values as our resultant outcomes: -

t_3*	Q*	TAC*
8.16	**152.2484**	**5789.6**

4.2 Numerical Example number 2

Should we consider a numerical illustration and employ select parameter values inherent to our inventory model: -
α= 5, c_1 = 30, c_2= 10, c_3 =10, κ=400 per unit time, t_1=1.5, t_2=3.5, λ=0.55, C_p=25 per unit, S_c=120 per unit, H_C=0.5 per unit time, D_C=10 per unit.
Subsequently, we insert these specific values into equations numbered (15) and (18). Employing the mathematical software MATLAB (R2021a), we tackle this problem, yielding the following optimal values as our resultant outcomes: -

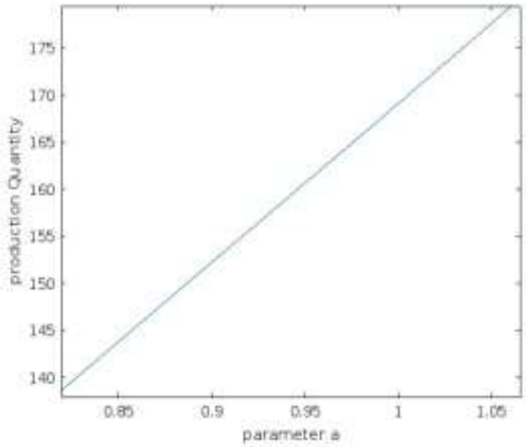
t_3*	Q*	TAC*
9.00	**226.2**	**10445**

5 Sensitivity analysis

For sensitivity analysis of this Model, we change values of parameters one by one and announce the effects on t_3*, Q* and TAC*. Rate of changes (in percentage) in values of parameters are taken -20 %, -10%, +10% and +20%.

Parameter	Change in Parameter	t_3*	Q*	TAC*
α =0.9	-20%	8.11	121.9	1820.2
	-10%	8.13	137.02	3807.5
	10%	8.18	167.43	7766.5
	20%	8.20	182.69	9738.5
C_1=50	-20%	8.49	161.64	2485.7
	-10%	8.31	156.94	4396.8
	10%	8.02	147.55	6939.1
	20%	7.90	142.85	7896.8
C_2 = 30	-20%	7.94	174.77	10851
	-10%	8.05	163.49	8383.7
	10%	8.27	141.00	3050.5
	20%	8.40	129.75	143.46
C_3=10	-20%	8.17	183.83	8902.1
	-10%	8.16	168.04	7346
	10%	8.15	136.45	4232.9

	20%	8.15	120.66	2675.9
	-20%	8.08	58.33	-



	-10% 10% 20%	8.12 8.19 8.23	105.28 199.20 246.16	2244.8 9318 12831
$t_1 = 1$	-20% -10% 10% 20%	8.15 8.15 8.16 8.16	152.24 152.24 152.24 152.24	5695.4 5741 5840.9 5894.5
$t_2=5$	-20% -10% 10% 20%	6.92 7.54 8.76 9.36	240.01 199.19 99.69 41.87	11406 8864.5 2225.3 -
$\lambda=0.95$	-20% -10% 10% 20%	9.08 8.71 8.00 6.72	224.49 194.5 141.21 63.24	- 1771.3 6849.2 3687
$C_P = 20$	-20% -10% 10% 20%	8.15 8.15 8.16 8.16	152.24 152.24 152.24 152.24	5714.9 5752.3 5826.9 5864.2
$S_C=100$	-20% -10% 10% 20%	8.16 8.16 8.16 8.16	152.24 152.24 152.24 152.24	5787.1 5788.4 5790.8 5792
$H_C=100$	-20% -10% 10% 20%	8.16 8.16 8.15 8.15	152.24 152.24 152.24 152.24	5056.8 5423.2 6156 6522.3
$D_C=50$	-20% -10% 10% 20%	8.16 8.16 8.15 8.15	152.24 152.24 152.24 152.24	5441.5 5615.5 5963.6 6137.6

6 .Graphs of Observations and Results

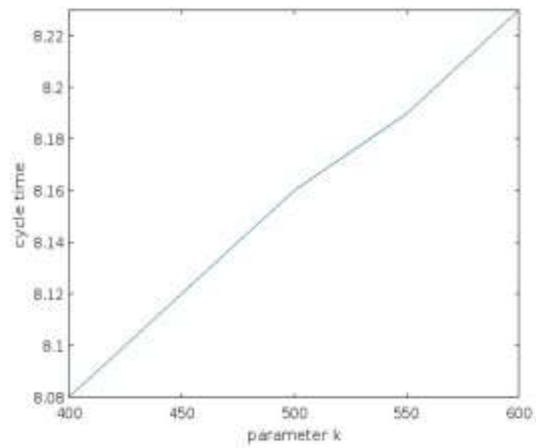
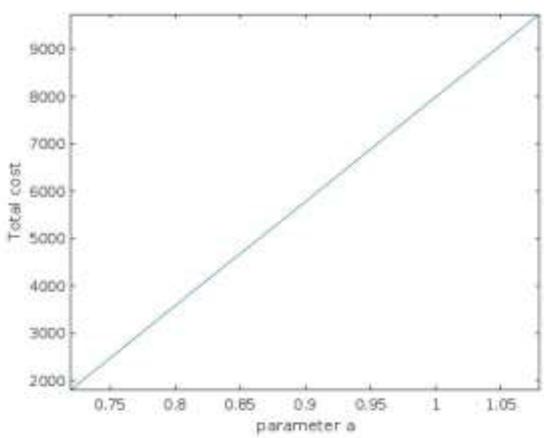


Figure 6.1

Figure 6.2



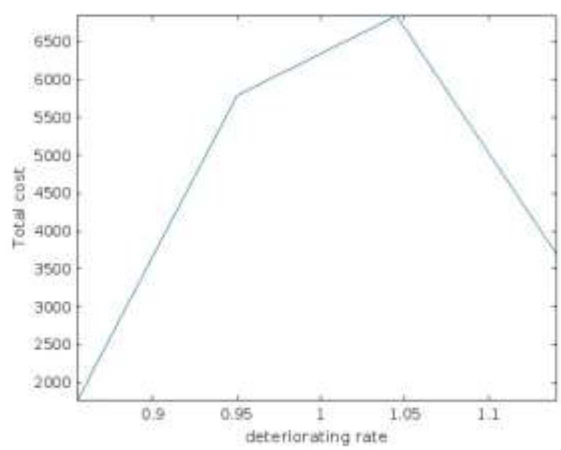
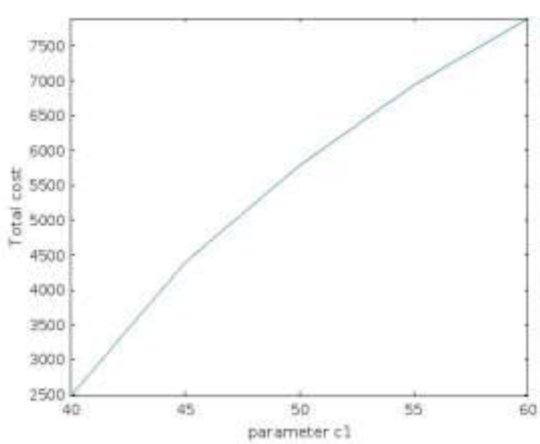


Figure 6.3

Figure 6.4



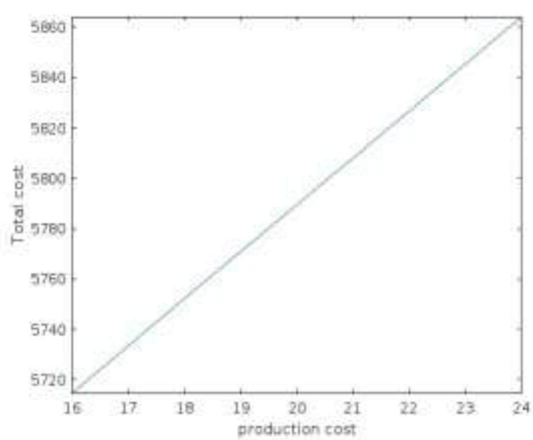


Figure 6.5Figure 6.6

7. Graphical Conclusion

Upon a comprehensive examination of the sensitivity analysis and a careful analysis of the graphs pertaining to our model, we deduce many results. We explain some of the following important conclusions from them:

(i) We observe that graph of figure 6.1 and find that Graphs of Optimum Total Cost is increasing faster in starting with respect to parameter C_1 and after some time increasing slowly, and after some time again increasing more slowly. So if we increase value of parameter C_1 then value of Total cost of Inventory system also increases.

(ii) In Figure 6.2, the graph depicting the Optimum Total Cost with respect to Production Cost exhibits a consistent upward trajectory. This signifies that as the production cost escalates, the total cost of the inventory system also experiences a proportional increase.

(iii) In Figure 6.3, the graph illustrating the relationship between the Optimum Production Quantity and the parameter "a" demonstrates a steady and uniform ascent. This indicates that as the value of parameter "a" is augmented, the Optimum Production Quantity of the inventory system likewise experiences a commensurate increase.

(iv) As per the data presented in Figure 6.4, the graphs depicting the relationship between Cycle Time and Parameter κ exhibit an initial consistent and gradual increase. Subsequently, there is a marginal uptick in the trend, followed by another phase of continuous and steady escalation. This pattern suggests that the influence of the parameter κ on the value of Optimum Cycle Time varies across distinct time intervals.

(v) Upon examination of Figure 6.5, it becomes evident that the graph depicting the relationship between the Optimum Total Cost and Parameter "a" exhibits a consistent and linear ascent. This observation signifies that as Parameter "a" is elevated, the Total Cost of the Inventory system experiences a proportional increase.

(vi) The graph depicting the Optimum Total Cost concerning the Deterioration Rate exhibits an initial phase of steady and consistent increase, resembling a straight-line trajectory. Subsequently, it experiences a marginal uptick for a certain duration, followed by a declining trend towards the end.

8 Novelty of this Article:

The primary impetus behind this issue lies in enhancing customer service within the manufacturing sector. This research is particularly advantageous to the retail industry, applicable to a wide range of products such as electronic components, fashionable attire, household goods, and various other items. The principal contribution of this paper resides in its comprehensive analysis, characterization, and derivation of inventory theory models. These models can be effectively employed to optimize inventory levels and, subsequently, enhance warehouse inventory management for representatives within the business sector.

9 Conclusion

In the present study, we have introduced a production inventory model for deteriorative goods. This model considers two distinct production levels. We assumed that the rate of item deterioration remains constant, while the demand rate follows a quadratic function over time. This scenario is favorable because it allows us to begin production at a lower rate, avoiding the accumulation of excessive initial inventory, which would otherwise lead to increased holding costs. This approach not only ensures customer satisfaction but also enhances the potential for profit. In this work, we have developed a mathematical model to address this situation and have provided a solution. To illustrate the applicability of the model, we have included a numerical example and conducted sensitivity analysis. The proposed inventory model can be a valuable tool for both manufacturers and retailers, helping them to accurately determine the optimal order quantity, cycle time, and overall inventory cost. For future research, this model could be extended in various directions, including accommodating different demand rate patterns like linear or cubic trends, considering with shortage. This model has been further expanded to accommodate multi-level production environments, encompassing cubical or quadratic demand patterns, as well as various forms of deterioration, including those that are time-dependent. Its capacity to generate insightful findings renders it applicable and valuable to a wide array of firms and industries.

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