Application of Burrxii-Dal and Weibull-Dal Distribution: A Case Study of Wheat Yield in Multan Pakistan

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Abstract

Background

Statistical analysis is highly dependent on the Probability distribution of the data and new models are most important part for the expansion of this field. Burr-XII DAL (BDAL) and Weibull-DAL (WDAL) distribution with five parameters are used to analysis the distribution and to predict the yield and growth rate of different products.

Objective

The purpose of the study is to determine the effectiveness of the application of modified BURRXII-DAL and WEIBULL-DAL distribution for the analysis of the wheat yield in Multan, Pakistan by estimating their maximum likelihood.

Methodology

The study involves the comparative analysis of Burr-XII DAL (BDAL) and Weibull-DAL (WDAL) distribution with other distribution models including Beta exponentiated Nadarajah Haghighi (BENH), Kumaraswamy Nadarajah Haghighi, exponentiated generalized Nadarajah Haghighi and modified Nadarajah Haghighi (MNH). The properties of Burr-XII DAL (BDAL) and Weibull-DAL (WDAL) analyzed through the R package and software to determine MLE, standard errors and W^*, A^*, K-S, p-value, AIC, BIC and $\hat{l}$ along with Akaike information criteria (AIC) and Bayesian information criteria (BIC).

Conclusion

The study concluded that modified propose Burr-XII DAL (BDAL) and Weibull-DAL (WDAL) are effective alternative models for the analysis of the distribution due to their high reliability and maximum likelihood value for the data make it reliable for the analysis of the real-life data.

Keywords: Burr-XII DAL (BDAL) distribution, Maximum Likelihood, Weibull-DAL (WDAL) distribution

Introduction

Statistical analysis of the data is performed through multiple models that help to predict the probability distribution and provide better understanding of the different available parameters and their function in the real life. The development of the innovative and creative distribution models that are highly accessible are the requirement of the time since 1980. This distribution model focused on production of skewed distribution, beta-generation, accumulation parameter techniques and combined technique.

Burr-XII distribution is the model that was developed by Burr (1942) having its expansive application in the field due to its 12 cumulative frequency functions. On the other hand, Weibull distribution is the continuous parameter probability distribution model that helps in the providing the analysis of the field data and physical properties of the data and hazardous including the failure time. Weibull, W. (1951). A statistical distribution function of wide applicability.

Zhang and Xie (2011) describe the bathtub shaped hazard function's features and presentation of the 2-paramter Weibull distribution. Similarly, Xie et al (2002) reported a three-parameter modified Weibull extension having tub shaped hazard function. Lai et al (2003) proposed modified Weibull (MW) distribution by multiplies the Weibull cumulative hazard function αx^{θ}bye$^{\lambda x}$, which was further modified to exponentiated form by Carrasco et al (2008). Bourguignon, M., Silva, R. B. and Cordeiro, G. M. (2014). The Weibull-G family of probability distributions.

Burr type XII distribution is described as type XII that helps in providing the software reliability by Abdel-Ghaly et al. (1997) while Wang and Keats (1996) focused on the likelihood technique of the Burr XII distribution for the detection of point and interval estimators of all variables. Zimmer et al. (1998) reported the statistical and probabilistic properties of Burr type XII distribution along Burr type XII distribution association to other distributions hat were used for the analysis of the reliability for every parameter of model. Ali Mousa and Jaheen (2002) described that Burr distribution have the Bayesian estimation of the parameters that supported progressively modified samples. Wu and Yu (2005) acclaimed essential quantities to check the form parameter and establish confidence interval of the form parameter of the Burr type XII distribution under the failure censored strategy. Dimitrikopoulou, D., Adamidis, K. and Loukas, S. (2007). A lifetime distribution with an upside-down bathtub shaped hazard function. Peña-Ramírez, F. A., Guerra, R. R., & Cordeiro, G. M. (2019). The Nadarajah-Haghighi Lindley distribution. Oluyede, B. O., Mashabe, B., Fagbamigbe, A., Makubate, B., & Wanduku, D. (2020). The exponentiated generalized power series: Family of distributions: theory, properties and applications. Anwar, M. and Bibi, A. (2018). The Half-Logistic Generalized Weibull Distribution. Weibull-DAL distribution with five parameters is discussed in the study of Shahbaz et al. (2021). Later, modification of the BurrXII-DAL and Weibull-DAL distribution with the help of five parameters was discussed by Shahbaz et al., (2021). They focused on the application of the proposed distributions by using the dataset for the waiting time of the bank customers. The case study for the factors related to the yield of rice was discussed by (Shahbaz et al., 2021) to highlight the properties of BurrXII-DAL and Weibull-DAL distribution with five parameters.

Yousof et al. (2017) suggested Burr XII distribution and Yousof et al. (2018) recommended a Weibull Generalized G intimate as the replacement of continuous models as these provide the positive shape parameters. Altun et al (2018) provide an alternative replacement of 4-parameter generation exemplary that descried as Odd Log-Logistic Burr XII distribution and each parameter was distinct and examined. The model has the mathematical properties, and each mathematical property are significant. The two truncated moments relationship is supported through the hazard function that was described through the Particular convenient categorization consequences, also the provisional anticipation of each variate was estimated and the proper explanation of convinced functions of each obtainable variate. The utmost likelihood technique is employed to assessment the model parameters concluded resources of a graphical Monte Carlo simulation study. Furthermore, Altun described the replacement of the log-location regression classic that was equally supported the suggested distribution.

The purpose of the study is to be identified and examined new modified models of distribution including Burr-XII DAL and Weibull-DAL. The other main aim of the study is to estimate each distribution parameters through assessing the maximum likelihood and efficient presentation of data through the graphical and mathematical method. The study also focused on the finding the most effective distribution models aby comparing the new modified models with other models and find the application of the new models into the real-life data for determining the yield of wheat in the Pakistan especially in Multan.

Material and Methods

The study involves the assessment of BXII-DAL and Weibull-DAL distribution by using the parameter models in which they were assessed by using R software. The study parameters were statistically analyzed by using maximum likelihood. The likelihood helped in providing the perceived information matrix. On the other hand, the fit of two sets of real data of the proposed distribution were analyzed through, by comparing its values with the three and two parameters. The shape parameter was also analyzed in which the unknown shape parameter approximation was analyzed and compared with other models shape by estimating the mean square errors. This helped in providing the estimates of reliability function that were associated through the assimilated mean squared error.

BXII-DAL distribution

The model parameters of BXII-DAL distribution were assessed by the using maximum likelihood strategy and four goodness-of-fit statistics for finding the better role in the real data. The proposed BurrXII-DAL model is associated with Kumaraswamy generalized power Weibull, Beta Exponentiated Nadarajah Haghighi , Exponentiated Generalized Power Weibull, Generalized Power Weibull and Nadarajah Haghighi distributions by using the two real data sets. This association helped in to describe the probability of the BXII-DAL model in comparison with other model distribution. To examine the usefulness of BXII-DAL model, study focused on exploring BXII-DAL AIC and the comparison of AIC as performed with all models. The BXII-DAL goodness of fit was determined by inserting the parameters and it provide the information that help to provide maximum likelihood of function of BXII-DAL model along with other models. Thus, MLE had a greater significance while standard errors of each model was estimated for each parameter value to further examine any variation among original and fitted values. Table 1.1 provides the MLEs of the model parameters while the values of the goodness of fit statistics for the fitted models were enumerated in Table 1.2. The densities of the competitive models were respectively, given by

$$f_{KwGPW}(x) = ab\alpha\beta\lambda x^{\beta-1}(1+\lambda x^{\beta})^{\alpha}\exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right)\left(1-\exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right)\right)^{\alpha-1} \times \left[1-\{1-\exp\left(1-[1+\lambda x^{\beta}]^{\alpha}\right)\}^{a}\right]^{b-1} \quad ; \ a,b,\alpha,\beta,\lambda > 0. \quad (i)$$

$$f_{BENH}(x) = \frac{\alpha\beta\lambda}{\beta(a,b)}\left(1+\lambda x^{\beta}\right)^{\alpha}\exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right)\left\{1-\exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right)\right\}^{\alpha\beta-1} \times \left[1-\left\{1-\exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right)\right\}^{\beta-1}\right]^{b-1} ss \quad ; \quad a,b,\alpha,\beta,\lambda > 0.(ii)$$

$$l(\theta) = n\log(\alpha\lambda) - n\log[B(a,b)] + \left(1-\frac{1}{\alpha}\right)\sum \log((1+\lambda x)^{\alpha}) + (a-1)\sum \log\left(1-e^{1-(1+\lambda x)^{\alpha}}\right) + b\sum 1-(1+\lambda x)^{\alpha}$$

The MLE for all parameters are included below by differentiating above equation.

$$U_a = \frac{\partial l}{\partial a} = -n\psi(a) + n\psi(a+b) + \sum \log\left(1-e^{1-(1+\lambda x)^{\alpha}}\right)$$

$$U_b = \frac{\partial l}{\partial b} = -n\psi(b) + n\psi(a+b) + \sum 1-(1+\lambda x)^{\alpha}$$

$$U_\alpha = \frac{n}{\alpha} - \frac{1}{\alpha^2}\sum \log((1+\lambda x)^{\alpha}) - \left(1-\frac{1}{\alpha}\right)\sum \frac{(1-(1+\lambda x)^{\alpha})^{\alpha}}{(1+\lambda x)^{\alpha}} + (1-\alpha)\sum \frac{(1-(1+\lambda x)^{\alpha})^{\alpha} e^{1-(1+\lambda x)^{\alpha}}}{1-e^{1-(1+\lambda x)^{\alpha}}} + b\sum (1-(1+\lambda x)^{\alpha})^{\alpha}$$

$$U_\lambda = \frac{\partial l}{\partial \lambda} = \frac{n}{\lambda} - \left(1-\frac{1}{\alpha}\right)\sum \frac{(1-(1+\lambda x)^{\alpha})^{\lambda}}{(1+\lambda x)^{\alpha}} + (1-a)\sum \frac{(1-(1+\lambda x)^{\alpha}) e^{1-(1+\lambda x)^{\alpha}}}{1-e^{1-(1+\lambda x)^{\alpha}}} + b\sum (1-(1+\lambda x)^{\alpha})^{\lambda}$$

$$f_{EGPW}(x) = \alpha\beta\lambda\theta x^{\beta-1}(1+\lambda x^{\beta})^{\alpha} \exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right)\left\{1-\exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right)\right\}^{\theta-1} \quad a,b,\alpha,\beta,\lambda > 0. \quad \text{(iii)}$$

$$f_{GPW}(x) = \alpha\beta\lambda x^{\beta-1}(1+\lambda x^{\beta})^{\alpha} \exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right) \quad ; \quad x,\alpha,\beta,\sigma > 0 \quad \text{(iv)}$$

$$f_{NH}(x) = \alpha\lambda(1+\lambda x)^{\alpha} \exp(1-[1+\lambda x]^{\alpha}) \quad ; \quad x,\alpha,\lambda > 0 \quad \text{(v)}$$

The study is proposed to evaluate and analysis the application of best-fitted model from all models. This analyzed through the production of wheat yield per acre from Multan. The data is obtained from the Pakistan Bureau of Statistics.

The data obtained from the sample consisted of 100 values that were described as:

0.75	**0.79**	**0.81**	**1.28**	**1.46**	**1.79**	**1.98**	**2.01**
2.32	2.44	2.68	2.91	3.12	3.41	3.56	3.78
3.90	4.12	4.21	4.33	4.34	4.33	4.51	4.54
4.72	4.81	4.89	4.93	4.98	5.01	5.21	5.32
5.52	5.65	5.73	6.12	6.32	6.34	6.54	6.65
6.76	6.92	7.32	7.4	7.5	7.64	7.87	7.99
8.12	8.27	8.38	8.47	8.56	8.87	8.99	9.0
9.4	9.54	9.67	9.88	9.45	9.88	10.3	10.5
10.7	10.82	10.98	11.03	11.12	11.4	11.5	12.3
12.7	12.9	13.4	13.5	13.7	13.8	13.9	14.1
14.6	14.9	15.3	15.6	15.9	17.3	17.8	18.4
18.6	19.1	19.6	20.5	21.7	21.9	24.3	27
28.2	29.5	29.8	30.4				

Data: Wheat Yield 2021

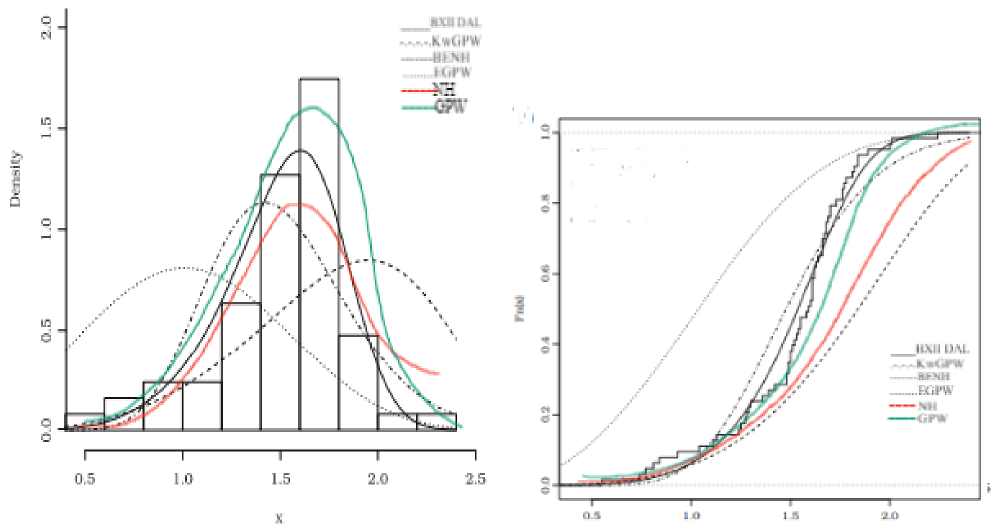
The following data represents the Wheat yield data that is retrieved from (Pakistan Bureau of Statistics, 2021). Table (1) provides the MLE values of the data that help in estimation of the model parameter for the data along with the corresponding Standard error SEs (given in parenthesis) and the goodness of fit statistics.

Table (1): *MLEs and their standard errors (in parentheses) for data.*

Distribution	a	b	α	β	λ	θ
BXII-DAL	0.4253 (0.0059)	1.8806 (0.0044)	1.7344 (0.1015)	1.2757 (0.4283)	0.7920 (0.0355)	0.4253 (0.0059)
KwGPW	3.1889 (0.0540)	0.2320 (0.0597)	0.7920 (0.2600)	2.9113 (0.1359)	- -	3.1889 (0.0540)
BENH	1.2757 (1.2514)	3.1889 (3.4718)	0.2320 (0.6218)	0.7920 (0.3270)	1.2757 (1.2514)	- -
EGPW	- -	- -	0.0433 (0.0246)	8.0992 (4.2118)	0.0035 (0.0039)	21.9928 (6.8421)
GPW	- -	- -	0.0083 (0.0037)	18.7243 (8.6113)	9.3543 (-)	- -
NH	- -	- -	2.2994 (0.2130)	- -	0.0020 (0.0001)	- -

Table (2): *The statistics analysis of data (W^*, A^*, K-S, p-value, AIC, BIC and $\hat{l}$):*

Distribution	W^*	A^*	K-S	p-value(K-S)	AIC	BIC	$\hat{l}$
BXII-DAL	0.326	0.840	0.227	0.437	600.892	612.989	321.396
KwGPW	0.324	0.927	0.304	0.525	617.975	621.528	271.987
BENH	0.4291	0.8325	0. 312	0.532	892.78	884.2554	439.394
EGPW	0.6628	0.2411	2.989	0. 8341	922.608	957.051	491.814
GPW	2.4215	2.3114	0.739	0.0000	968.65	956.89	837.325
NH	0.1526	2.3469	0.2919	0.0000	800.11	807.94	398.05



(a) (b)

Figure (1.1) *Plots of estimated pdf (a) and cdf (b) of BXII-DAL for data*

Weibull-DAL distribution

The following section of Weibull-DAL, described fits of the W-DAL of wheat yield means and data sets is compared with the other models including Weibull Dagum , Weibull power function , Weibull Lomax , generalized power Weibull, Nadarajah Haghighi, Beta Weibull and Kumaraswamy Weibull distributions with the wheat yield. The competitive models had the densities that were described below, respectively,

$$f_{WDa}(x) = ab\alpha\beta\lambda x^{\beta-1}\left[\frac{\left(1+\lambda x^{-\beta}\right)^{-ab-1}}{\{1-(1+\lambda x^{-\beta})^{-\alpha}\}^{b+1}}\right] exp\left[-a\left\{\frac{\left(1+\lambda x^{-\beta}\right)^{-\alpha}}{1-(1+\lambda x^{-\beta})^{-\alpha}}\right\}^{b}\right] \quad (i)$$

$$a, b, \alpha, \beta, \lambda, x > 0.$$

$$f_{WPF}(x) = ab\alpha^{-\beta}\beta x^{\beta-1}\left[\frac{\left\{\left(\frac{x}{\alpha}\right)^{\beta}\right\}^{b-1}}{\left\{1-\left(\frac{x}{\alpha}\right)^{\beta}\right\}^{b+1}}\right] exp\left[-a\left\{\frac{\left(\frac{x}{\alpha}\right)^{\beta}}{1-\left(\frac{x}{\alpha}\right)^{\beta}}\right\}^{b}\right] \quad , \quad a, b, \alpha, \beta, x > 0. (ii)$$

$$f_{WLx}(x) = ab\alpha\beta^{-1}(1+\frac{x}{\beta})^{-\alpha-1}\left[\frac{\left\{1-\left(1+\frac{x}{\beta}\right)^{-\alpha}\right\}^{b-1}}{\left\{\left(1+\frac{x}{\beta}\right)^{-\alpha-1}\right\}^{b+1}}\right] exp\left[-a\left\{\frac{1-\left(1+\frac{x}{\beta}\right)^{-\alpha}}{\left(1+\frac{x}{\beta}\right)^{-\alpha}}\right\}^{b}\right] \quad , iii)$$

$$a, b, \alpha, \beta, x > 0.$$

$$f_{GPW}(x) = \alpha\beta\lambda x^{\beta-1}\left(1+\lambda x^{\beta}\right)^{\alpha} exp\left(1-\left[1+\lambda x^{\beta}\right]^{\alpha}\right) \quad , \quad x, \alpha, \beta, \lambda > 0 \ (iv)$$

$$f_{NH}(x) = \alpha\lambda(1+\lambda x)^{\alpha} exp(1-[1+\lambda x]^{\alpha}) \quad , \quad x, \alpha, \lambda > 0. (v)$$

$$f_{BW}(x) = \frac{\alpha\beta x^{\beta-1} exp\left(-\alpha x^{\beta}\right)}{\beta(a,b)}\left[1-exp\left(-\alpha x^{\beta}\right)\right]^{\alpha-1}\left[exp\left(\alpha x^{\beta}\right)\right]^{b-1} \quad , vi)$$

$$x, a, b, \alpha, \beta, \lambda > 0$$

$$f_{KwW}(x) = ab\alpha\beta x^{\beta-1} exp\left(-\alpha x^{\beta}\right)\left(1-exp\left[-\alpha x^{\beta}\right]\right)^{a-1}\left[1-\left\{1-exp\left[-\alpha x^{\beta}\right]\right\}^{a}\right]^{b-1} \ (vii)$$

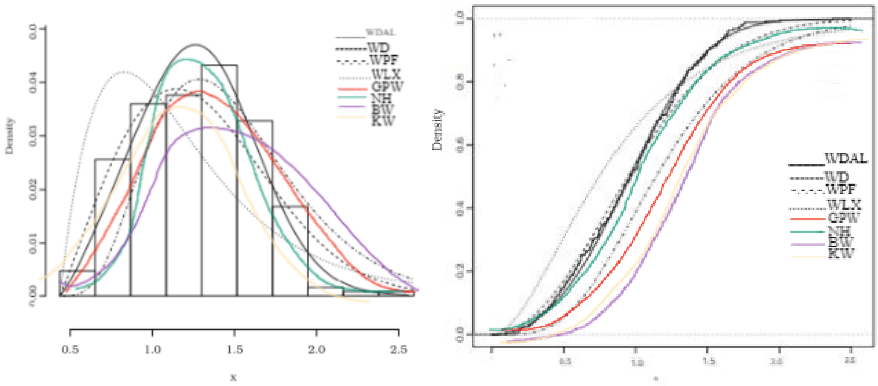
$$x, a, b, \alpha, \beta > 0$$

Table 3: *MLEs and their standard errors:*

Distribution	α	β	a	b	λ
W-DAL	0.0864	17.4410	0.5548	0.4513	0.5733
	((0.0153)	(0.2821)	(0.2542)	(0.0401)	(0.4230)
BW	1.3116	0.4310	0.8373	4.7270	-
	(2.3456)	(4.3563)	(0.0114)	(0.3810)	-
KwW	2.9113	3.1889	0.0403	0.7511	-
	(0.2600)	(0.0597)	(0.3988)	(0.0020)	-
WDa	0.1196	1.3778	0.0403	0.7920	1.3778
	(0.0020)	(0.0540)	(0.3988)	(0.3988)	(0.2600)
WPF	0.1196	0.7511	0.8412	0.1094	-
	(0.2600)	(0.0597)	(0.3988)	(0.0020)	-
WLx	0.8412	0.1094	1.3778	0.1196	-
	(4.0503)	(3.4718)	(0.0540)	(0.1359)	-
GPW	0.126	0.740	-	-	0.0403
	(0.0020)	(0.3988)	-	-	(0.0540)
NH	0.106	-	-	-	0.1196
	(0.2600)	-	-	-	(0.0597)

Table (4): *The statistics analysis of data (W^*, A^*, K-S, p-value, AIC,BICand$\hat{l}$):*

Distribution	W^*	A^*	K-S	p-value (K-S)	AIC	BIC	$\hat{l}$
W-DAL	3.1174	0.5684	0.106	0.393	651.9031	652.103	251.396
BW	1.0059	0.1821	.6543	.0610	834.0610	840.083	236.1830
KwW	1.7572	0.3210	.8182	0.5078	990.5078	998.390	293.1898
WDa	1.1833	0.8240	1.3778	0.1196	918.792	919.899	251.396
WPF	0.106	0.638	0.106	0.393	887.831	814.661	250.916
WLx	0.144	0.917	0.124	0.215	927.975	912.528	251.987
GPW	0.206	0.438	0.106	0.393	987.831	914.661	250.916
NH	0.344	0.917	0.124	0.215	827.975	812.528	251.987



(a) (b)

Figure (2): *Plots of estimated pdf (a) and cdf (b) of W-DAL.*

Result:

The aim of the current study is to find out the application and usage of BXII-DAL and W-DAL model and its distribution and application in the real-life data for the study of the yield of the wheat in Multan, Pakistan. Table (1) shows that the MLE & Standard error of BXII-DAL model with other comparative models which shows that BXII-DAL is better than previous models for wheat yield data set having **a** 0.4253, **b** 1.8806, **α** 1.7344, **β** 1.2757, **λ** 0.7920 and **θ** 0.4253.

Furthermore, in table (2) the value of BXII-DAL statistic AIC, A^* and W^* is smallest among the fitted models. AIC of **BXII-DAL** model is reduced from the earlier estimated value of 968.65 to the final value of 600.892. Moreover, BIC of BXII-DAL model is reduced from the earlier estimated value of 957.051 to the final value of 612.989, that showed that there is the significant improvement in the overall model fit of BXII-DAL as compared to other models. Hence, the current study concluded that the proposed BXII-DAL Model is best and most efficient model among the fitted models for the analysis and estimation of the data in the real life.

The Figure (1.1) shows of BXII-DAL distribution describe the graphical representation of probability distribution function and cumulative distribution function of proposed model. The graph shows that data for wheat yield is almost negatively skewed which is well predicted by BXII-DAL while distributions of other models do not well exhibit the data and that is very difficult to predict the distribution. The graph also described CDF and it is concluded that wheat yield is over estimated by EGPW model while according to NH model the wheat yield is under estimated and this estimation is highly described by BXII-DAL. The graphical plots that are generated through the BXII-DAL distribution is unimodal, bimodal and is very flexible to model distributions that described through different shapes including right skewed, left-skewed, symmetrical (approximately) and reversed-J. The graphical plots for the hazard rate of BXII-DAL distribution that described through the different shapes including bathtub, upside-down bathtub, decreasing, and increasing. The moments of BXII-DAL distribution indicated shape, size, peak, and spread of distribution by skewness, kurtosis, mean and variance and all these features described that the peaks of the graphs are actually due to increased values of parameters and this parameter increased is not observed by smaller parameters of the models. Further, smaller values of parameters generate increase smooth upward of the plots that describe the increase in the HRF.

The Table (3) of the current study shows that the MLE & Standard error of W-DAL model is better than previously used models with other comparative models for wheat yield data set having a 0.0864, b 17.4410, α 0.5548, β 0.4513, λ 0.5733 and θ 0.5733.

Furthermore, in table (4) the value of W-DAL statistic AIC, A^*, W^*, and K-S is smallest among the fitted models as compared to the other models. AIC of W-DAL model is reduced from the earlier estimated value of 990.5078 to the final value of 651.9031. Moreover, BIC of W-DAL model is reduced from the earlier estimated value of 998.390 to the final value of 652.103. the overall analysis of the statistic described that there is the significant improvement in the overall model fit as compared to the other fittest model. The overall analysis of the W-DAL model described that the proposed W-DAL Model is best and efficient model among the fitted models.

The Figure (2) of W-DAL model shows the graphical representation of probability distribution function and cumulative distribution function of the wheat yield data. The data for wheat yield shows that there is the almost negatively skewed graph, and this is well predicted by W-DAL while distributions of other models do not well exhibit the data through their graphical representation. The graph also described for CDF wheat yield is over estimated by EGPW model while NH model shows under estimated values of wheat yield that is effectively represented by W-DAL. The Weibull-DAL showed Hazard rate function as unimodal distribution that is completely flexible, and its flexible changes were shown through the distribution shape, and this is obtained through the changing the parameters which enables the innovative model considerable extra applied to appropriate statistics sets under several shapes. The hazard rate of Weibull-DAL describing the failure rate is constant over time for a decreasing function while on the other hand the increased failure rate is described through the right skewed distribution and directly related for negatively skewed distributions.

Conclusion

The proposed and modified BurrXII-DAL (BDAL) and Weibull-DAL (WDAL) distribution are considered as the most effective alternative models for describing the distributions due to their reliability, flexibility, estimation through the maximum likelihood of the parameters and fitting of data for goodness. All these features made these models as the more effective models for their application to real life data like in the wheat yield.

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