HELLICALLY CORRUGATED WAVEGUIDE FILLED WITH PLASMA

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Abstract

In this paper, we discuss the effect of plasma on helically corrugated waveguide. Specifically, the next section deals with synthesis of dispersion relation of helically corrugated waveguide completely filled with an ideal plasma.

Introduction

Vacuum microwave tubes are the leading generators of electromagnetic energy for the applications in which medium or high power is required. Although conventional microwave tubes to be suitable for many of these applications, the performance of a large number of systems, which is determined by a combination of the bandwidth, efficiency, and microwave power, is limited by the capabilities of conventional vacuum microwave tubes. Research has showed that the presence of controlled amount of ionized gas (plasma) inside microwave tubes can improve the characteristic beyond what is currently available in evacuated devices. In particular, the presence of plasma can significantly increase the frequency bandwidth, operating efficiency, and power handling capabilities of microwave tubes and allow operation without a guiding magnetic field. This can be attractive for systems and applications where size and weight are important **[1-5].**

Since a plasma consist of charged particles (electron and ions), it is subject to numerous natural oscillations in the microwave frequency range. For this reason, a device filled with plasma may serve as a source of microwave radiation.

The presence of the plasma changes both the beam propagation conditions and the electromagnetic properties of plasma filled tubes. Plasma influence on the electromagnetic characteristics of SWS's can be described by the parameter $\left(\omega_p/\omega\right)$, as long as the plasma density is not too high $\left(\omega_p \le \omega\right)$. Here ω_p and ω are the electron plasma frequency and the frequency of the vacuum circuit mode. The plasma in this case is

expected to up shift the electromagnetic branches of the vacuum modes approximately as $\omega_0^2 = \omega_p^2 + \omega^2$, where ω_0 is the up shifted frequency of the plasma-filled circuit. It is observed that plasma filling changes the electromagnetic properties of SWS in two manners: 1) the frequency of the electromagnetic modes is upshifted and 2) a plasma mode often referred to as the Trivelpiece-Gould mode is created **[6-11-]**.The electrostatic modes of a plasma filled periodic structure are quite complex and form a dense spectrum **[12-13].**

1.2 Dispersion relation of plasma filled helically corrugated waveguide

Consider a perfectly conducting helically corrugated waveguide completely filled with ideal electron plasma. The radius profile of the waveguide is similar to the chapter 2. Wall radius

$$R(z,\theta) = R_0 + h\cos(k_0 z + q_0\theta) \qquad (1)$$

Where $k_0 = \frac{2\pi}{z_0}$ and $q_0 = \frac{2\pi}{\varphi_1}$.

Periodicity in axial direction is while periodicity in azimuthal direction is φ_1. Here R_0 is the mean radius of the waveguide, h is the amplitude of the corrugation, and z_0 is the corrugation period.

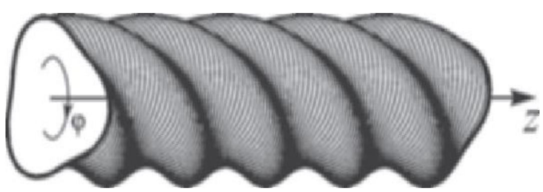

Fig1 **Schematic view of a waveguide with a three-fold right-handed (q_0=3) helical corrugation.**

In case of plasma filled waveguide

$$\vec{\nabla}\times\vec{E} = \frac{i\omega}{c}\vec{B} \qquad (2a)$$

$$\vec{\nabla}\times B = \frac{4\pi\vec{J}}{c}\vec{E} - \frac{i\omega}{c}\vec{E}$$

$$\vec{J} = \sigma\vec{E} \quad \text{and} \quad \sigma = \frac{i\omega_p^2}{4\pi\omega}$$

$$\vec{\nabla}\times B = -\frac{i\omega}{c}\left[1 - \frac{\omega_p^2}{\omega^2}\right]\vec{E}$$

$$\vec{\nabla}\times B = -\frac{i\omega}{c}\varepsilon.\vec{E} \qquad (2b)$$

Where $\varepsilon = \left(1 - \frac{\omega_p^2}{\omega^2}\right)$

Solving above equations (2a) and (2b) for transverse and axial components of electric and magnetic fields. Substituting the transverse components for E_z *and* B_z and rearranging the two equations we get the well known Bessel equation.

The solution of the equation representing axial fields is therefore

$$E_z = AJ_l\left(\beta r\sqrt{\varepsilon}\right)e^{i\varphi} \quad \text{and}$$
$$B_z = AJ_l\left(\beta r\sqrt{\varepsilon}\right)e^{i\varphi}$$

where

$$\varphi = -(\omega t - k_z z - l\theta)$$

Using above equations we can write

$$\vec{E} = \frac{e^{i\varphi}}{\beta^2}\begin{bmatrix} ik_z\beta\sqrt{\varepsilon}\frac{\partial}{\partial\left(\beta r\sqrt{\varepsilon}\right)}J_l\left(\beta r\sqrt{\varepsilon}\right)A - \frac{lk}{r}J_l(\beta r)D \\ -\frac{lk_z}{r}J_l\left(\beta r\sqrt{\varepsilon}\right)A - ik\beta\frac{\partial}{\partial(\beta r)}J_l(\beta r)D \\ \beta^2 J_l\left(\beta r\sqrt{\varepsilon}\right)A \end{bmatrix}$$

(3a)

and

$$\vec{B} = \frac{e^{i\varphi}}{\beta^2}\begin{bmatrix} \frac{lk}{r}J_l\left(\beta r\sqrt{\varepsilon}\right)A + ik_z\beta\frac{\partial}{\partial\beta r}J_l(\beta r)D \\ ik\beta\sqrt{\varepsilon}\frac{\partial}{\partial\beta r\sqrt{\varepsilon}}J_l\left(\beta r\sqrt{\varepsilon}\right) - \frac{lk_z}{r}J_l(\beta r)D \\ \beta^2 J_l(\beta r)D \end{bmatrix}$$

(3b)

FIELDS IN HELICAL WAVEGUIDE

Due to Floquet periodicity both in azimuthal as well as axial direction, we can write the azimuthal as well as axial fields as

$$\vec{E} = \sum_{n=-N}^{n=N}\sum_{p=-N}^{p=N}\frac{e^{i\phi_{np}}}{\beta_n^2}\begin{bmatrix} ik_{zn}\beta_n\sqrt{\varepsilon}\frac{\partial}{\partial\beta_n r\sqrt{\varepsilon}}J_{l_p}(\beta_n r\sqrt{\varepsilon})A_{np} & -\frac{l_p k}{r}J_{l_p}(\beta_n r)D_{np} \\ -\frac{l_p k_{zn}}{r}J_{l_p}(\beta_n r\sqrt{\varepsilon})A_{np} & -ik\beta_n\frac{\partial}{\partial\beta_n r}J_{l_p}(\beta_n r)D_{np} \\ \beta_n^2 J_{l_p}(\beta_n r\sqrt{\varepsilon})A_{np} & \end{bmatrix}$$

(4)

$$\vec{B} = \sum_{n=-N}^{N}\sum_{p=-N}^{N}\frac{e^{i\varphi_{np}}}{\beta_n^2}\begin{bmatrix} \frac{l_p k}{r}J_{l_p}(\beta_n r\sqrt{\varepsilon})A_{np} & ik_{zn}\beta_n\frac{\partial}{\partial\beta_n r}J_{l_p}(\beta_n r)D_{np} \\ ik\beta_n\sqrt{\varepsilon}\frac{\partial}{\partial\left(\beta_n r\sqrt{\varepsilon}\right)}J_{l_p}(\beta_n r\sqrt{\varepsilon})A_{np} & -\frac{l_p k_{zn}}{r}J_{l_p}(\beta_n r)D_{np} \\ & \beta_n^2 J_{l_p}(\beta_n r)D_{np} \end{bmatrix}$$

(5)

where

$$\varphi_{np} = -(\omega t - k_{zn}z - l_p\theta),\ k_{zn} = k_z + nk_0 \ \text{and}\ l_p = l + pq_0$$

Here N is an integer tending to infinity.

Boundary Conditions

$$E_z - hk_0\sin\left(k_0 z + q_0\theta\right)E_r\Big|_{R(z,\theta)=R_{00}+h\cos(k_0 z+q_0\theta)} = 0$$

(6a)

$$E_\theta + \alpha\cos\left(k_0 z + q_0\theta\right)E_\theta - q_0\alpha\sin\left(k_0 z + q_0\theta\right)E_r\Big|_{R(z,\theta)=R_0+h\cos\left(k_0 z+q_0\theta\right)} = 0$$

(6b)

Where $\alpha = \frac{h}{R_0}$

Substituting electric field components E_r, E_θ from (4) in the first and second boundary conditions we get

$$\sum_{n=-N}^{n=N}\sum_{p=-N}^{p=N}\frac{e^{i(nu+p\delta)}}{\beta_n^2}\left\{\begin{matrix} \beta_n^2 J_{l_p}\left(\beta_n R\sqrt{\varepsilon}\right)A_{np} - \frac{hk_0\beta_n}{2}\sin\left(u'\right) \\ \begin{bmatrix} ik_{zn}\sqrt{\varepsilon}\left(J_{l_p-1}\left(\beta_n R\sqrt{\varepsilon}\right) - J_{l_p+1}\left(\beta_n R\sqrt{\varepsilon}\right)\right)A_{np} \\ -k\left(J_{l_p-1}\left(\beta_n R\right) + J_{l_p+1}\left(\beta_n R\right)\right)D_{np} \end{bmatrix}\end{matrix}\right\} = 0$$

(7a)

$$\sum_{n=-N}\sum_{p=-N}^{N}-\frac{e^{i(nu+p\delta)}}{\beta_n^2}\left\{\begin{array}{l}\frac{\left[1+\alpha\cos(u')\right]}{2}\left[\begin{array}{l}k_{zn}\beta_n\sqrt{\varepsilon}\left(J_{l_p-1}\left(\beta_n R\sqrt{\varepsilon}\right)+J_{l_p+1}\left(\beta_n R\sqrt{\varepsilon}\right)\right)A_{np}\\+ik\beta_n\left(J_{l_p-1}(\beta_n R)-J_{l_p+1}\beta_n R\right)D_{np}\end{array}\right]+\\\frac{q_0\alpha}{2}\sin(u')\left[\begin{array}{l}ik_{zn}\beta_n\sqrt{\varepsilon}\left(J_{l_p-1}\left(\beta_n R\sqrt{\varepsilon}\right)-J_{l_p+1}\left(\beta_n R\sqrt{\varepsilon}\right)\right)A_{np}\\-k\beta_n\left(J_{l_p-1}(\beta_n R)+J_{l_p+1}(\beta_n R)\right)D_{np}\end{array}\right]\end{array}\right\}=0$$

(7b)

DISPERSION RELATION

$$\sum_{n=-N}^{N}\sum_{p=-N}^{N}\frac{4\pi}{\beta_n^2}\left\{\begin{array}{l}\left[\beta_n^2 R_{l_p}^{nm}-\frac{hk_0\beta_n}{4}k_{zn}\sqrt{\varepsilon}\left(P_{l_p}^{nm-1}-P_{l_p}^{nm+1}-Q_{l_p}^{nm-1}+Q_{l_p}^{nm+1}\right)\right]A_{np}\\-\frac{ihk_0k\beta_n}{4}\left[\left(H_{l_p}^{nm-1}-H_{l_p}^{nm+1}+L_{l_p}^{nm-1}-L_{l_p}^{nm+1}\right)\right]D_{np}\end{array}\right\}\delta_{p-o-n=m,0}=0$$

(8)

$$-2\pi\sum_{n=-N}^{N}\sum_{p=-N}^{N}\frac{1}{\beta_n^2}\left\{\begin{array}{l}\left[\begin{array}{l}k_{zn}\beta_n\sqrt{\varepsilon}\left(P_{l_p}^{nm}+Q_{l_p}^{nm}\right)+\frac{\alpha.k_{zn}\beta_n\sqrt{\varepsilon}}{2}\left(P_{l_p}^{nm-1}+P_{l_p}^{nm+1}+Q_{l_p}^{nm-1}+Q_{l_p}^{nm+1}\right)\\+\frac{q_0\alpha}{2}k_{zn}\beta_n\sqrt{\varepsilon}\left(P_{l_p}^{nm-1}-P_{l_p}^{nm+1}-Q_{l_p}^{nm-1}+Q_{l_p}^{nm+1}\right)\end{array}\right]A_{np}\\+i\left[\begin{array}{l}k\beta_n\left(H_{l_p}^{nm}-L_{l_p}^{nm}\right)+\frac{\alpha.k\beta_n}{2}\left(H_{l_p}^{nm-1}+H_{l_p}^{nm+1}-L_{l_p}^{nm-1}-L_{l_p}^{nm+1}\right)\\+\frac{q_0\alpha}{2}k\beta_n\left(H_{l_p}^{nm-1}-H_{l_p}^{nm+1}+L_{l_p}^{nm-1}-L_{l_p}^{nm+1}\right)\end{array}\right]D_{np}\end{array}\right\}\delta_{p-o-n=m,0}=0$$

(9)

Equations (8) and(9), will give the dispersion relation

We denote

$$i=(2N+1)[N+m]+N+o+1$$
$$j=(2N+1)[N+n]+N+p+1,$$

Then (.9) and 10) can be written as

$$\sum_{j=1}^{(2N+1)(2N+1)}\left(T_{ij}G_j+U_{ij}K_j\right)=0 \qquad (10a)$$

and

$$\sum_{j=1}^{(2N+1)(2N+1)}\left(V_{ij}G_j+W_{ij}K_j\right)=0 \qquad (10b)$$

where

$$T_{ij}=\left[R_{l_p}^{nm}-\frac{hk_0k_{zn}\sqrt{\varepsilon}}{4\beta_n}\left(P_{l_p}^{nm-1}-P_{l_p}^{nm+1}-Q_{l_p}^{nm-1}+Q_{l_p}^{nm+1}\right)\right]\delta_{p-o-n+m,0}$$

(11a)

$$U_{ij}=\left[-\frac{ihkk_0}{4\beta_n}\left(H_{l_p}^{nm-1}-H_{l_p}^{nm+1}+L_{l_p}^{nm-1}-L_{l_p}^{nm+1}\right)\right]\delta_{p-o-n+m,0}$$

(11b)

$$V_{ij}=\frac{k_{zn}\sqrt{\varepsilon}}{\beta_n}\left[\begin{array}{l}\left(P_{l_p}^{nm}+Q_{l_p}^{nm}\right)+\frac{\alpha}{2}\left(P_{l_p}^{nm-1}+P_{l_p}^{nm+1}+Q_{l_p}^{nm-1}+Q_{l_p}^{nm+1}\right)\\+\frac{q_0\alpha}{2}\left(P_{l_p}^{nm-1}-P_{l_p}^{nm+1}-Q_{l_p}^{nm-1}+Q_{l_p}^{nm+1}\right)\end{array}\right]\delta_{p-o-n+m,0}$$

(11c)

$$W_{ij}=\frac{ik}{\beta_n}\left[\left(H_{l_p}^{nm}-L_{l_p}^{nm}\right)+\frac{\alpha}{2}\left(H_{l_p}^{nm-1}+H_{l_p}^{nm+1}-L_{l_p}^{nm-1}-L_{l_p}^{nm+1}\right)+\frac{q_0\alpha}{2}\left(H_{l_p}^{nm-1}-H_{l_p}^{nm+1}+L_{l_p}^{nm-1}-L_{l_p}^{nm+1}\right)\right]\delta_{p-o-n+m,0}$$

(11d)

Equation (10a) and (10b) are joined to a matrix form given by

$$\begin{bmatrix}T_{ij} & U_{ij}\\V_{ij} & W_{ij}\end{bmatrix}\begin{bmatrix}G_j\\K_j\end{bmatrix}=0 \qquad (12)$$

In order to have a nonzero set of G_j and K_j, the following determinant must be zero, namely

$$D(\omega,k) = \det\begin{bmatrix} T_{ij} & U_{ij} \\ V_{ij} & W_{ij} \end{bmatrix} = 0 \qquad (13)$$

The required dispersion relation for helically corrugated waveguide can be given by the above matrix equation (14).

The value of T_{ij} and V_{ij} will change in comparison to the cold dispersion relation of HRW. Equation (13) is the relation of mean hybrid modes that have both rf axial magnetic and electric field components in the plasma filled helically corrugated waveguide with applied axial magnetic field of finite strength.

Results

The result is that in contrast to the vacuum electromagnetic mode, hybrid modes with strongly increased amplitude of the axial component of the electric field near the axis of the SWS can occur, thus allowing a strong coupling. In plasma-filled device the interaction can involve both electromagnetic modes of the SWS and electrostatic modes supported by the plasma.

References

[1] G. Burt *et al.*, *Phys. Rev. E*, vol. 70, 2004.
[2] Graeme Burt, Sergy V.Samsonov] G. G. Denisov, V. L. Bratman, A. W. Cross, W.He,A.D. R. Phelps, K.Ronald, S. V.Samsonov, IEEE Trans. Plasma Sci., vol. 33, no.2, pp. 661–667April., 2005.
[3] S. V. Samsonov *et al.*, Phys. Rev. Lett. , vol. 92, p. 118 301, 2004.
[4] R.A.Bromlry, B.E.Callan, ,proc.IEEE 114 (9) 1213-1218. (1967).

[5] S. V. Samsonov, V. L. Bratman, G. G. Denisov, M. Y. Glyavin, V. K.Lygin, A. G.Luchinin, Y. V. Bykov, A. A. Bogdashov, and M. Thumm,*IEEE Trans. Plasma Sci.* vol.32, No. 6, pp. 884–889, Dec.2004.
[6] G.S. Nuinovich, Y. Carmel, T.M. Antonsen, D.M.Goebel and J.Santoru, IEEE Trans. Plasma Sci., vol. 26, No. 3, pp. 628-645, June 1998.
[7] A.W.Trievelpiece and R.W.Gould, J.Appl. Phys. Vol.30. pp. 1784-1799, 1959.
[8]M.V.Kuzelev,O.T.Loza,A.A.Rukhadze, P.S.Strelkov and A.G.Shkvarunets, Plasma Phys. Reports,vol.27 No.8, pp. 669- 691, 2001.
[9] D. Bohm and E. Gross, Phys. Rev; vol. 75, pp 1851- 1876,1949.
[10] Y. Carmel, K. Minami, R .A Kehs, W. W. Destler, V.L.Granatstein, D.Abe, and W.L.Lou,, Phys. Rev; vol.62, pp 389-2392, 1989.
[16] [11] Y.Carmel, W.R.Lou, T.M. Antonsen Jr., J.Rodgers B.Levush, W.W.Destler, and V.L.Granatstein,Phys.Fluids; vol. 4, pp 2286-2292, 1992.
[12] G.Shkvarunets, S.Kobayashi, J.Weaver, Y.Carmel, J.Rodgers,T.M. Antonsen Jr., V.L.Granastein,Destler W W.W "IEEE Transactions on Plasm a Science vol. 24 pp. 905-917 June 1996
[13] Preeti vyas, Arti Gkhale Y.Choyal, and K.P.Maheswari Pram Pranma Journal of Ph ys. Indian academy of science.Vol.56 pp. 625-633 may 2001.