RES MILITARIS
REVUE EUROPEENNE D ETUDES EUROPEAN JOURNAL OF MILITARY STUDIES
Social Science Journal

Some results of resolvability ideal space

By

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Abstract

We found mathematical concepts in ideal space that led to the partitioning of space into three pairwise disjoint: * - Frontier set, disputed set and the outer function. These sets have a direct and indirect relationship to the *- resolvable and *- irresolvable spaces. Also, we put a new type of open sets, we called them I_R-Open and I^*_R-open and the relationship of the topologies of generates with urysohn space, we also included some important results of resolvability ideal spaces.

Introduction

Hewi H E. [4] in 1943 was the first to discuss and establish the necessary and sufficient condition for determining the space that can be resoved into two CD-Sets, as well as relating these spaces with semi – regular spaces, as the first to define regular open sets is M.H. stone [8] in 1937. In 1960 -1966 Kuratowski [7] and Vaidyan athaswamy [10] developed a definition synonymous with the limit points using the concept of idealism, where they called all these points the local function to a certain set. Using the local function, Vaidyana [12] put the concept of the ψ- operator. Almohammed [1] studied this ψ- operator in the different forms in the fuzzy ideal spaces. AL.Talkany [9 , 2] studied this ψ- operator in the proximity i- topological spaces . AL-Rubaye [3] studied the separation axiom via specied case of local function. Julian, Maximilian and David [6] defined the *-dense as follows G is *-dense, if G^*=X and the space is called *-resolvable, if the space divides into two sets *-denses. The researcher Y.K. Aftalkany [13] invereted the proximity i- topological spaces to define the density which call it congested set and studied its most important properties. In 2019 the searchers Selim. Modak and Islam [11], which defined the frotier set in ideal space as follow $Fr*(Q) = Q*\cap(X-Q)^*$, for any subset Q of X.

Definition 2.1: Let Q be a subset of ITS (X, T , I):

- The outer function of Q is defined by $O(Q) = \{x \in X; \exists\, U \in \tau(x) \ni U \cap A \in I\} = \psi(X-Q) = X - Q^*$
- The disputed of Q is defined by $d(Q) = Q^* \cap \psi(Q)$
- The *- Frontier [11] of Q is defined by $Fr^*(Q) = Q^* \cap (X-Q)^*$

From it we conclude that the universal set X is decomposition into three pairwise disjoint, are

*-frontier set, disputed set and outer function $\ni X = Fr^*(Q) \cup d(Q) \cup O(Q)$, any subset Q of X . Also, we see that $Q^* = \psi(Q) \cup Fr^*(Q)$

- For I-dense Q subset of X is divide the space X into two disjoint set $Fr^*(Q)$ and d(Q) and $Fr^*(Q) = X - \psi(A), O(Q) = \emptyset , d(Q) = \psi(Q)$
- If X is *-resolvable, then d(Q) = $\emptyset$ for any subset Q of X.
- If $\tau \cap I \neq \emptyset$, then $\psi(Q) \neq \emptyset$, for any subset Q of X.
- $O(Q \cup P) = O(Q) \cap O(P)$.
- $O\big(O(Q)\big) = \psi(Q^*)$.
- O(Q)$\subset O(Q^*) = O(X - O(Q))$.
- For Q$\subset P \rightarrow O(P) \subset O(Q)$.
- d(Q)$\cup\, d(P) \subseteq d(Q \cup P)$.
- $d(Q \cap P) \subseteq d(Q) \cap d(P)$.
- $Q^* = X - O(A)$.
- If X is *-resolvable, then
- $Fr^*(Q) = X$ for some $Q \subseteq X$
- $d(Q) = \emptyset$ and $O(Q) = \emptyset$
- For any open set U with condense I, $U^* = X$.
- If (X, τ , I) contains an isolated point, then X is *-irresolvable

Definition 2.2: Let (X, τ , I) be any ITS, we call the set Q is *- nowhere dense, if $O\big(O(A)\big) = \emptyset$

A mong the properties that are accepted and do not require much effort to prove

- $O\big(O(A)\big) = \psi(A^*)$.
- $for\ any\ ITS\ (X ,\tau, I)\ and\ Y \subseteq X$, if $H \subset Y$ is *- nowhere dense in X it's *- nowhere dense in Y.
- If I is condense and H is *-nowhere dense in Y, then it's *-nowhere dense in X.

Definition 2.3: Let (X, τ , I) be any ITS, and $Q \subseteq X$ we call the set Q is:

- I – open [4], iff $Q \subseteq Int(Q^*)$, the family of all I- open subset of X denoted by I.O(X)
- I_R- open iff $Q = Int(Q^*)$, the family of all I_R- open subsets of X denoted by I_R. O(X).

From the above definitions, we conclude that:

- Each I_R- open is open, but the converse is not true always, if we take X= {x, y, z} , τ={ $\emptyset$, X, {x}, {y, z} and I = { $\emptyset$, {x}, {y}, {x, y} } , {x} is open but not I_R- open .
- The family I.O(x) is base for some topology on X and the topology generated by its, we denoted by τ_I .
- I_R. O(X) family generate subbase and the topology generated by its, we denoted by τ_{IR}.
- Each I- open + k- open (Q=Int (cl(Q)) ⟶ I_R- open.
- If I is condense ⟶ R – open = I_R- open.

So, each urysohn I_R- space is urysohn space ⟶ (X, τ_{IR}) is T_2- space.

- Each dense + I_R- open set Q ⟶Q(X-Q) is nowhere dense.
- If Q is I_R- open + *- dense ⟶ Q^* is * - dense. □ If Q^* is *- dense ⟶ Q, is I – open + *- dense.

Definition 2.4: Let (X, τ , I) be any ITS, we call that:

- A point $x \in X$ has empty ψ- tightness if whenever $x \in K^* \ni H \subset K \ni x \in H^*$ and

$\psi(H) = \emptyset$.

- * - dense K subset of X, whose complement is also *- dense is said to be I_{CD}- set in X

Proposition **2.5:** Every *- resolvable space has empty ψ- tightness.

By *- resolvablity of X, these will be two disjoint subsets H, K of X meet the expectations, $\psi(H) = \psi(K) = \emptyset$. For any $x \in M^* \rightarrow (M \cap K) = \emptyset, \psi(M \cap H) = \emptyset \; and \; x \in M^* = (M \cap K)^* \cup (M \cap H)^*$ so, either $x \in (M \cap K)^*$ or $x \in (M \cap H)^*$. that means x has empty ψ- tightness.

Note: If $X - \{x_0\}$ is *- dense, that $X - \{x_0\}$ is not τ^*- closed

Proposition 2.6: (X, τ , I) is *- resolvable iff every non – empty open subset of X containing set which is * - resolvable in its relative topology.

Let $\emptyset \neq M \in \tau$, since X is * - resolvable $\exists \; D^* = (X - D)^* = X$ and $M \cap (X - D)^* \subseteq (M \cap (X - D))^*$, $M \cap D^* \subseteq (M \cap D)^* \rightarrow M \cap D, M \cap (X - D)$ and I_{CD}- sets in Y . Contrariwise, let $\emptyset \neq G_0 \in \tau \rightarrow \exists \; H_0 \subset G_0$ is * - resolvable $\exists \; D_0, E_0$ are disjoint I_{CD}- set in $H_0 \ni H_0 \subseteq (D_0 \cap H_0)^*$, $H_0 \subseteq (E_0 \cap H_0)^*$. put $G_1 = X - H_0^* \in \tau \rightarrow \exists H_1 \subseteq G_1$ is * - resolvable $\rightarrow \exists D_1, E_1$ are disjoint I_{CD} - set in $H_1 \ni H_1 \subseteq (D_1 \cap H_1)^*$, $H_1 \subseteq (E_1 \cap H_1)^*$, suppose that H_σ with I_{CD}- sets D_σ, E_σ in H_σ, $\forall \sigma < \lambda, \lambda$ also being an ordinal number $G_0 \supset G_1 \supset \cdots \supset G_\lambda$. If $G = X - (\cup_{\sigma<\lambda} H_\sigma)^* \neq \emptyset \rightarrow \exists \; H_\lambda \subset G$ is * - resolvable, D_λ, E_λ are I_{CD}- set in H_λ. By induction $\exists$ some ordinal number λ such that $G_0 \supset G_1 \supset \cdots \supset G_\lambda \supset \cdots \supset (X - (\cup_{\sigma<\lambda} H_\sigma)^*) = \emptyset$ with $D = \cup_{\lambda<\Delta} D_\lambda, E = \cup_{\lambda<\Delta} E_\lambda$ since $\cup_{\lambda<\Delta} H_\lambda \subseteq \cup D_\lambda^* \subset (\cup_{\lambda<\Delta} D_\lambda)^* \rightarrow D^* = X$ similarly $E^* = X$ and $D \cap E = \emptyset$. Therefore, X is * - resolvable.

Definition 2.7: A set G is said to be I_R^*- open if $G = \psi(G^*)$.

- Note that the intersection of any two I_R^*- open sets is I_R^*- open.
- Ideal space is said to be semi I_R^*- regular if its I_R^*- open subsets from a subbase for all open sets and the topology generated by its we denoted by $\tau_{I_R^*} \subset \tau$.
- If $G \in \tau_{I_R^*} \rightarrow X - G^* \in I_R^*$.
- $\forall$. $G \in \tau_{I_R^*} \rightarrow G^* = G^*(\tau_{I_R^*})$
- If $G \in I_R^* \rightarrow G = \psi(G^*(\tau_{I_R^*}))$.
- If $U \in \tau \rightarrow \psi(U^*)$ is I_R^*- open with X=X*.

From the above properties, we can get the following result

Proposition 2.8: Let (X, τ , I) be urysohn space and X=X* then, (X, $\tau_{I_R^*}$) is T_2- space.

Definition 2.9: ideal space is called:

1- SII- space if having the property that no subset of X is * - resolvable.

2- MII – space, if having the property that every *-dense subset of X is open.

Proposition 2.10: every MII- space is SII- space.

It is plain that every SII- space is * - irresolvable if possible $\exists$ a subset P which is * - resolvable $\rightarrow \exists CD - sets \; P_1, P_2 \; in \; P \ni P = p_1 \cup p_2, p_1 \cap p_2 = \emptyset, p \subset (P_1 \cap P)^*$, $p \subseteq (P_2 \cap P)^* \rightarrow P^* \subset P_1^{**} \subset P_1^* \rightarrow X = P^* \cup X - P^* \subset (P_1 \cup (X - P^*))^*$ $\rightarrow P_1 \cup P^{*c}$ is * - dense, so it's open, then

$\forall x \in P \ \exists Q$ is nbd of $x \ni Q \subset P_1 \cup (X - P^*)$
$\rightarrow Q \cap P \subset (P_1 \cup (X - P^*)) \cap P = P_1 \rightarrow Q \cap P \cap P_2 = \emptyset$. Thus P_2 is not *- dense relative to P. This contradiction. Then X is SII – space.

Proposition 2.11: A space (X, τ , I) containing a * - dense subset S which is * - resolvable in the relative topology is also * - resolvable.

Proof: if S is * - dense and * - resolvable $\rightarrow \exists S_1 \subset S \ni S \subset S_1^* \Rightarrow X = S^* \subset S_1^{**} \subset S_1^* \rightarrow S_1^* = X$ and $\psi_s(S_1) = \emptyset \rightarrow \psi_X(S_1) \cap S = \emptyset \rightarrow \psi_X(S_1) \cap S_1 = \emptyset \Rightarrow S_1 \subset (X - S_1)^* \rightarrow (X - S_1)^* = X$. Thus, X is * - resolvable.

Proposition 2.12: Let (X, τ , I) be condense MII – space, then every non-empty subset Q of X its $Q \in \tau_{Q^*}$

Proof: since $X - Q^* \in \tau \rightarrow X - Q^* \subset (X - Q^*)^*$ and $X = Q^* \cup X - Q^* \subset Q^* \cup (X - Q^*)^*$ so $Q \cup X - Q^*$ is * - dense $\rightarrow Q \cup X - Q^*$ is open.

Now, if $Q \notin \tau_{Q^*} \rightarrow \exists x \in Q \rightarrow x \in Q \cup X - Q^*$ and $\forall U \in \tau(x) \ni U \cap Q^* \cap X - Q \neq \emptyset$ $\rightarrow (Q \cup X - Q^*) \cap (Q^* \cap X - Q) \neq \emptyset$. This contradiction, because $X - Q^* \cap X - Q$ $(Q \cup X - Q^*) =$

.

Conclusion

An integrated study can be mode about the properties of the aggregates that are defined here I_R-open and I_R^*- open. As well as studing and identifying some of the topological concepts using these sets, such as compactness, separation axioms, connectedness, density and others. And also finding new definitions for functions of various shapes and types, continuous open, close, compact and other functions that correspond to those topological properties of them can be studied.

Reference

Almohammed. R, AL-Swidi L. A "New concepts of fuzzy local function" Baghdad Science Journal 17(2), pp. 515-522, 2020.

Al-Talkany, Y.K.M, Al-Swidi L.A.A.H. " On Some types of Proximity ψ – set" Journal of physics conference series. 1963(1), pp. 012-076, 2021.

Al-Swidi ,. L.A, Al-Rubaye. M. S. " New Classes of Separation Axiom Via Special case of Local Function" International Journal of Math Analysis, 8(21-24), pp. 1119-1131, (2014).

D.Jankovic and T.R>Hamlett "Compatible extensions of ideals" Bollettion U.M.I. Vol. 7 1992, pp. 453-465.

E. Hewitt " A Problem of set theoretic topology " Duke Math. J. 10, pp. 309-333, 1943.

Julian D., Maximilian G. and David R. " Ideal Resolvability " Topology APPI. 93, pp. 1-16. (1999).

K. Kuratowski " Topology" Vol. I, New York Academic Press. (1966).

M. H. Stone " Applications of the theory of Boolean rings to general topology “. Transactions of the American Math. Society, Vol 41. pp. 375-481 (1937).

Mahdi Altalkany , Y.K. , Al-Swidi L.A.A, " Focal Function in i- topological spaces via Proximity space" Journal of physics conference series 1591 (1) , 2020 .

R. Vaidyana tharwamy "Set Topology " Chelsea publishing, New York. (1960).

S. Selim, S. Modak and M.M. Islam " Characterizations of Hayashi- Sammuel Spaces Via boundary points “. Communication in advanced math. Sciences, Vol. II No. 3 , pp. 219226 , 2019 .

T. Nathaniec " on I-continuty and I-semi continuity points " Mathematica slovaca , 36(3) , pp. 297-312 (1986).

Y. K. Al-talkany and L.A. Al-Swidi " The Proximity congested set and not congested set" IOP Conference series, Journal of physics , 1999(1) , 2021.