

Vibration And Ride Comfort Analysis of Railway Vehicle System Subjected to Deterministic Inputs

By

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Abstract

Evaluation of vibrational behavior and ride quality/ride comfort is a crucial task for the analysis of ride comfort in railway dynamics as the railway vehicle is a complex system having many degrees of freedom, carbody have structural flexibility, wheel-rail interaction is function of non-linear parameters. The work investigates the effects of bump and pothole track irregularities on railway vehicle vibrational behavior and investigates the vehicle ride comfort index using Sperling method.

Keywords: Sperling ride index, railway vehicle, ride comfort, Lagrangian method, bump input

1. Introduction

Certain performing indicators like as riding quality, safety, contour negotiating and management, etc., are used to evaluate a highway or railways vehicle's overall effectiveness. Eulerian configurations of the objects, kinetic energies of rotation and translation, spring potential energies, the general forces, linear and angular velocity vectors, Rayleigh's dissipation energy, potential energy and Position vectors are all evaluated in the "Lagrangian formulation of the vehicle model" [1-8]. A coach on a train is made up of a vehicle-body, two bogie frames, and four wheels. Suspension components link these inflexible bodies together. Secondary suspension components separate the vehicle-body from the bogie frames, whereas primary suspension elements separate the bogie frame from the wheel axle [9-16].

In today's world railway vehicle-track designers are striving to achieve higher vehicle speed. The higher velocity creates a boost in the vibrational response in the carbody, which deteriorates the ride comfort, thus it requires altering the vehicle construction, or revising of

the suspension elements [17-24]. When the passive or semi-active suspension system efficiency is not satisfactory, the vehicle designer opts for the active suspension [25-33]. Though effective, the active suspension is not a worldwide choice, because of the reasons that the cost of operation and maintenance of this system is so high as compared with the benefits [34-41]. A weight reduction is a usual approach in designing railway vehicles to achieve higher speeds with less energy consumption. This weight reduction is subjected to lesser ground vibrations and reduced construction cost [42-49].

There have been several studies performed by the researchers for the evaluation of ride comfort of railway vehicles utilizing different approaches, different types of suspensions etc. Facchinetti et al [50] modelled secondary air spring suspension in railway vehicles for the analysis of safety and ride comfort. Kumar et al incorporated a human biodynamic model and seat characteristics along with the vehicle model and evaluated the ride comfort using Bond graph/Simulink approach [51]. Zhou et al determined the effects of carbody bounce flexibility on the ride quality of passenger railway vehicles [52].

In the present work, a 27 DoF rail vehicle model is formulated using Lagrangian method. The bump and pothole inputs have been considered from the track. The response of the carbody and bogie frame is studied when the vehicle is simulated to run at different speeds. The vehicle ride comfort is evaluated using Sperling Ride index (ISO 2631) at different simulated speeds.

2. Literature Review

Sharma et al., 2022 [1] presented a interdisciplinary analytical approach for determining the fatigued lifespan of the automobile shell construction of a railways locomotive and buggies subjected to randomized dynamical stresses. Initially, the dynamical force records of the vehicle-body architecture were analysed using a composite wear assessment approach, which included Multi-Body Component computation and the "Finite Element Method" (FEM). Dynamical strain is determined by applying longitudinally trains kinematics to the stress. Secondly, vehicle-body structural endurance lifespan and fatigued degradation were estimated using a complex degradation accumulating approach in wear assessment. Considerable consistency is found between the simulated outcomes from the analytical model and the investigational findings. After the FEM has been created, the mode may be calculated. The modal behaviour is employed as stimulation to generate the required kinetic load at every junction. Destruction to the vehicle's structure was calculated using a hybrid of the "NMCCMF" degradation accumulation approach and kinetic load. Therefore, author look at how the longitudinal stress affects the fatigue damage to the automobile structure. Fatigue degradation to the vehicle-body is exacerbated by the longitudinal stress.

Gopala et al., 2022 [7] taken into account a "quarter-car" design of a four-wheel automobile having "nonlinear" springs moving at a consistent rate on a randomized track condition with a previewing management to find the best automobile responsiveness. The "Bouc-Wen" approach assumes the nonlinear springy behaviour to be "hysteresis". The empirical interpolation method is used to this "nonlinear" automobile simulation. With the use

of the spectrum breakdown approach, we can calculate the "root mean square values" of the controlling forces, absorber movement, and roadway retaining attributes. Validation through "Monte Carlo" modelling of the corresponding linearized prototype derived from the SDM demonstrates that preview management improves automobile performance.

Sharma et al., 2021 [3] analysed the parameters of an "Indian Railway Rajdhani" (LHB) coach. The Lagrangian approach is used to create an appropriate computational technique with degree of freedom of 40. Eleven weight framework including passenger chair aid, position, carriage chassis, bolsters, bogie structure, and wheel axles are considered in the computational framework of a "rail vehicle". On a tangential course, the car is modeled moving at a velocity of 100 kilometres per hour. Testing information is gathered from the "research designs and standards organisation" (RDSO), Lucknow, and compared to the computation findings for verification. Statistical assessment is used to calculate an approximation of the influence that various rail-vehicle characteristics have on the riding experience.

Palli et al., 2021 [6] studied the dynamical responsiveness of a component underneath the operation of certain broad time-dependent stresses may be determined by a method known as time dependent dynamical assessment. The reloading period range is long enough that the inertial or absorption impacts are taken into account. The current study utilises finite element analysis to undertake a period - record assessment of a conventional locomotives carriage under Indian railway circumstances. After modelling a "bump in the track's surface" in the shape of an "ellipsoid" and assuming that an automobile travelling at that speed would cross the bump in 0.144 seconds, the authors mapped the time-dependent alteration in movement at several crucial regions on frameworks of trucks and cars subjected to the same loading settings. Because of the uneven weight arrangement, the forward and back of the locomotives wagon and carriage chassis are more sensitive to wheel resonance than the middle.

3. Mathematical Modeling

The present model is formulated using rigid body assumptions. The main rigid bodies considered are carbody, two bogie frames and four-wheel axle sets. Each rigid body is assigned vertical and lateral DoF. Carbody and bogie frames are further assigned angular DoF in three mutually perpendicular i.e., longitudinal, lateral and vertical directions. The wheel axle sets are further assigned angular DoF in i.e., longitudinal and vertical directions. A linear suspension is assumed between the different rigid bodies of the vehicle (Figure 1 and Figure 2). A linear creepage is assumed at the wheel-rail interface. The wind drag forces are not accounted for in the analysis. The static parts of the normal loads are evaluated using quasi-static analysis. The track is assumed flexible in vertical and lateral directions.

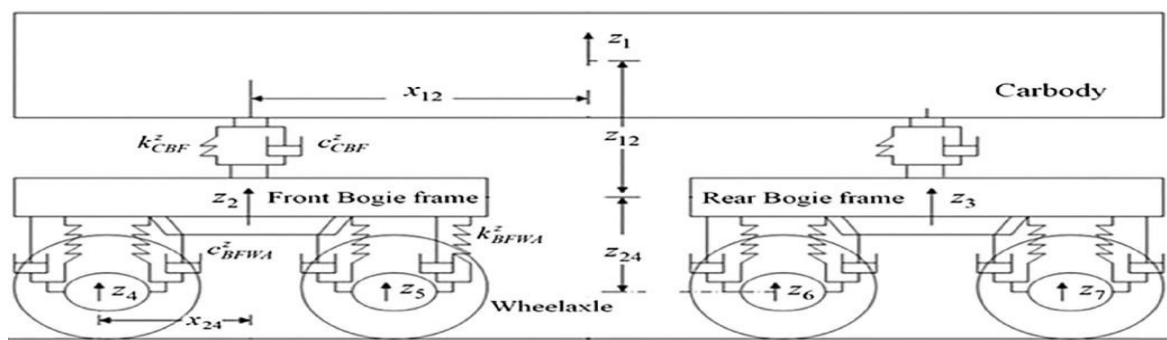


Figure 1. Vehicle model (Front View)

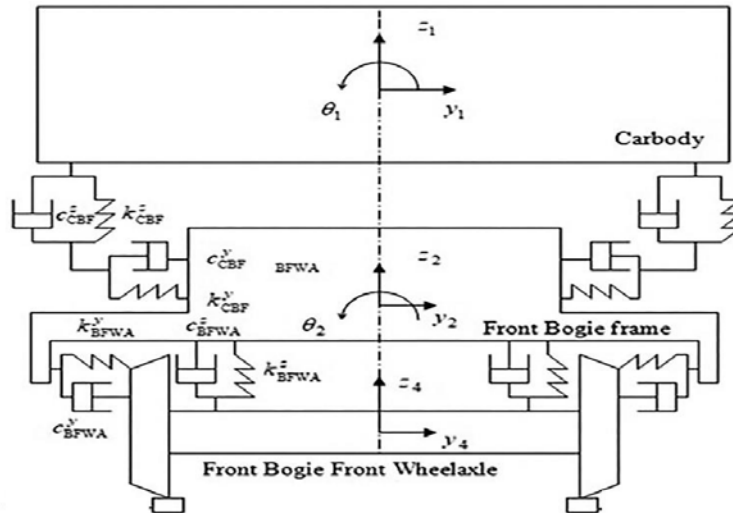


Figure 2. Vehicle model (Side View)

3.1. Equations of motion

The EoM of the formulated model is derived from Lagrange's approach and they may be expressed as:

$$\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{q}_i} \right] - \frac{\partial L}{\partial q_i} + \frac{\partial E_P}{\partial q_i} + \frac{\partial E_D}{\partial \dot{q}_i} = Q_i \tag{1}$$

3.2. Description of Track input

• **Bump input**

Bump road profiles are commonly observed on Indian roads for breaking the speed of vehicles. The mathematical model for finding out the coordinates of the bump is found by discretizing the bump as shown in Figure 3.

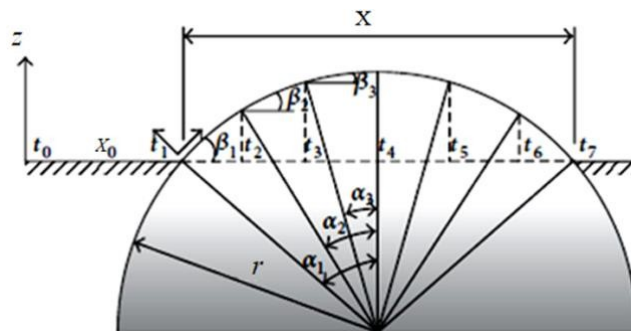


Figure 3. Discretization of circular bump profile

After discretization of the bump profile, relations between the length of bump (X), the radius of bump (R) and angle (α) of each element are as follows:

$$\left(\frac{X^2}{4} + Z^2 \right) \cdot \cos^2 \alpha - \frac{X^2}{2} \cdot \cos \alpha + \left(\frac{X^2}{4} - Z^2 \right) = 0 \tag{2}$$

Meanwhile, $d = \cos \alpha$

$$R = \frac{Z}{1 - \cos \alpha} \tag{3}$$

The values of time (t) and vertical displacement (y) for considered points, when the vehicle is running at a uniform speed V (m/s). Using Equations 3-12, time values are plotted on the abscissa and displacement coordinates are plotted on the ordinate (Figure 4). These values are obtained at a velocity of 40 kmph.

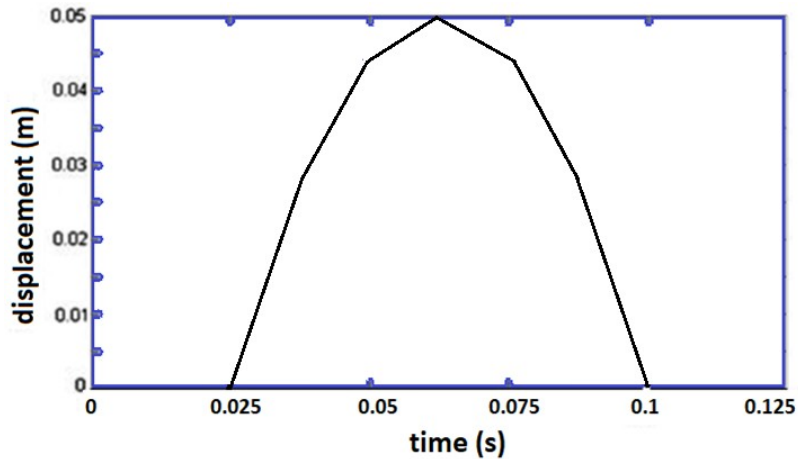


Figure 4. Bump road excitation

- **Pothole input**

Pot-hole Road input is formulated are same as bump road excitation except that y-coordinates are plotted in the negative direction (Figure 5).

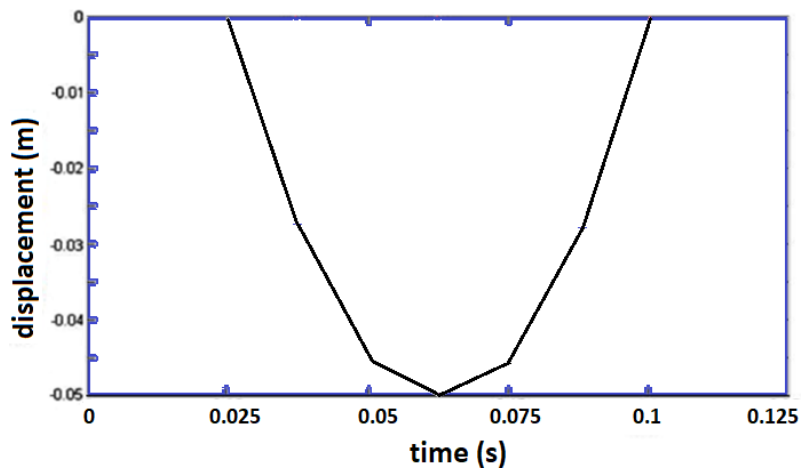


Figure 5. Pothole Road excitation (Simulink).

4. Results

The simulation is performed in MATLAB/SIMULINK environment. The vehicle is simulated to run at the velocity of 15 m/s and 60 m/s to observe the influence of velocity on kinematic parameters. Figure 6 shows vertical carbody acceleration in time domain for vehicle speed of 15m/s and 60m/s, which suggest that the higher speed increases the frequency of vibrations. Figure 7 shows vertical carbody acceleration in frequency domain for vehicle speed of 15 m/s and 60 m/s. At 15 m/s a peak acceleration of 8 m/s² is observed at the frequency around 8 Hz, meanwhile, a peak acceleration of 2 m/s² is observed at the frequency around 16 Hz. Figure 8 shows the front bogie frame acceleration in time domain for vehicle speed of 15 m/s and 60 m/s. The value of peak acceleration in bogie frame is higher at 60 m/s as compared with the same at 15 m/s. Figures 9 and 10 describe the carbody displacement under bump and pothole inputs which are useful when different types of suspensions are attached to the system and a comparative analysis is performed.

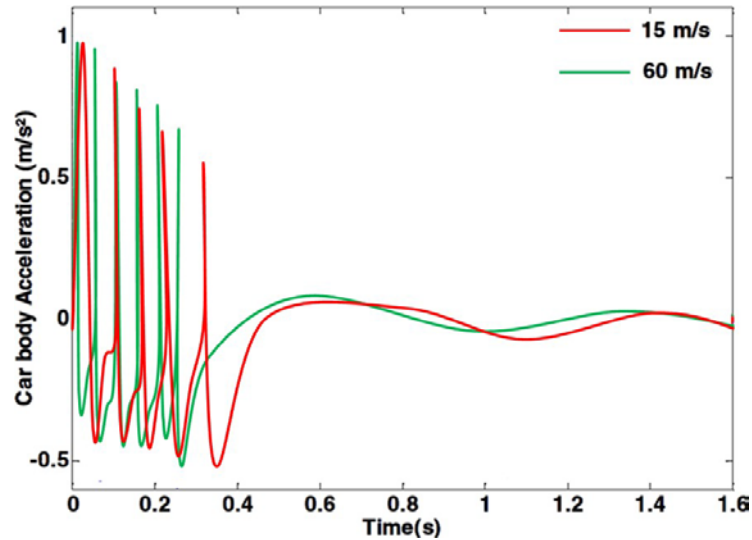


Figure 6. The vertical carbody acceleration in the time domain for the considered speeds

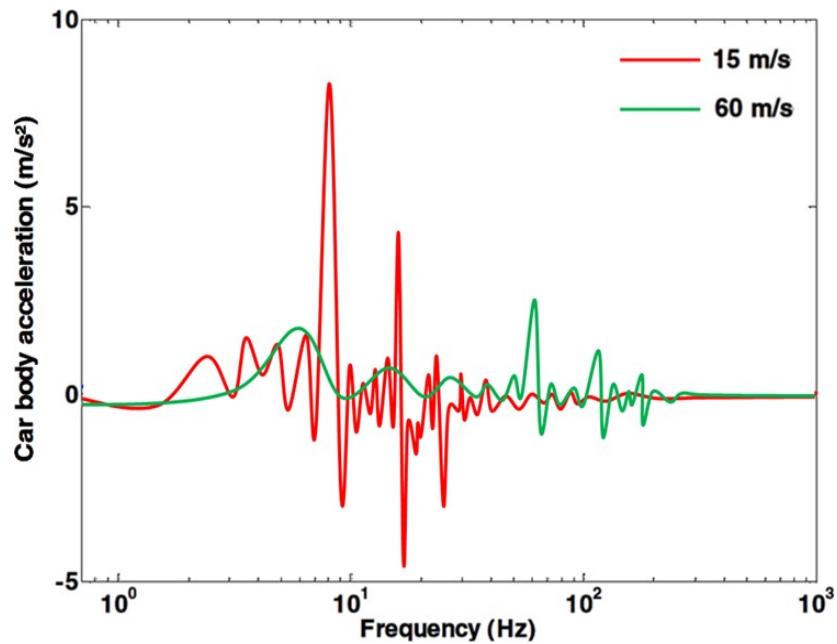


Figure 7. The vertical carbody acceleration in the frequency domain for the considered speeds

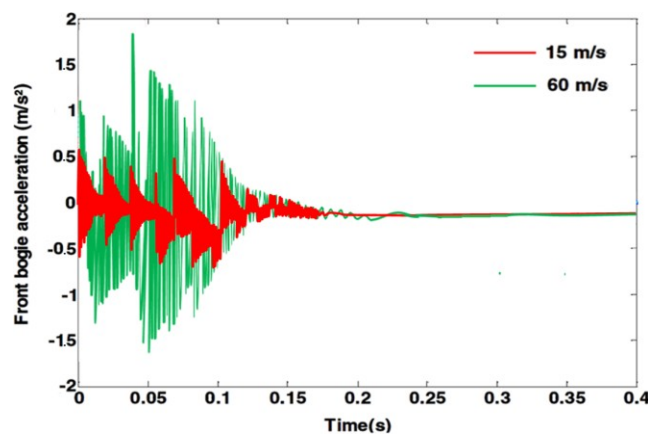


Figure 8. The front bogie frame acceleration in the time domain for the considered speeds

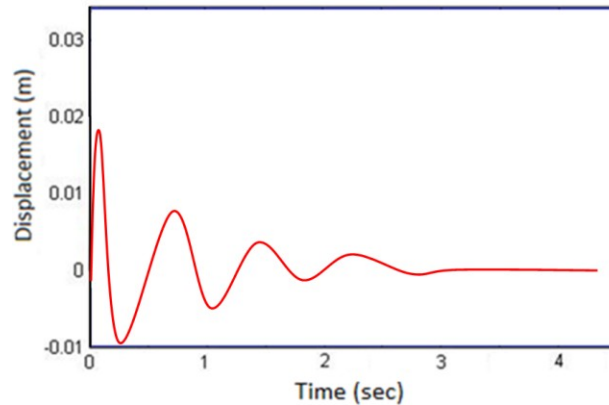


Figure 9. Carbody displacement under bump input

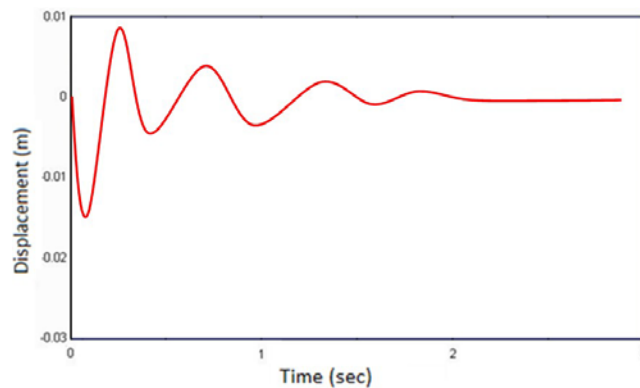


Figure 10. Carbody displacement under pothole input

4.1. Ride comfort

Sperling’s ride index is expressed as:

$$W_z = \left(\sum_{i=1}^{n_F} W_{z_i}^{10} \right)^{\frac{1}{10}} \tag{4}$$

W_z comfort index concerning the discrete frequency is expressed as:

$$W_z = \left(a_i^2 B(f_i)^2 \right)^{\frac{1}{6.67}} \tag{5}$$

Meanwhile a_i denotes the maximum peak acceleration response (in cm/s^2) and $B(f_i)$ is weighting factor, expressed as:

$$B(f) = k \sqrt{\frac{1.911f^2 + (0.25f^2)^2}{(1 - 0.277f^2)^2 + (1.563f - 0.0368f^3)^2}} \tag{6}$$

Table 1. Vehicle ride comfort index at considered speeds

Speed (m/s)	Sperling Index (W_z)
20	2.5
40	2.0
60	1.8

At the speed of 20 m/s the ride index is obtained as 2.5 which lies in the category of "More pronounced but not unpleasant". At the speed of 40 m/s the ride index is obtained as 2 which lies in the category of "clearly noticeable" (Table 1). At the speed of 60 m/s the ride index is obtained as 1.8 which lies in the category of “just noticeable”. The results show that ride index value decreases as the speed of the vehicle increases as shown in Figure 11.

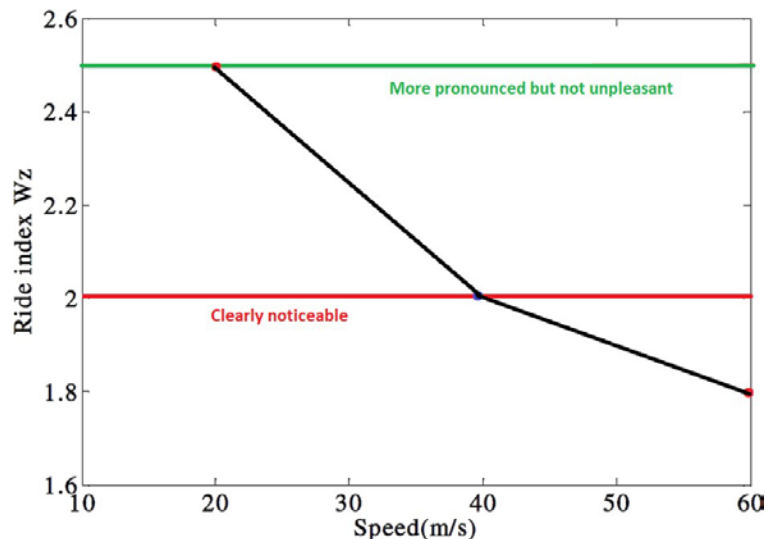


Figure 11. *Sperling's ride index at considered speeds*

5. Conclusions

In the present work, a 27 DoF railway vehicle model is formulated using Lagrangian method. The bump and pothole inputs have been considered from the track. The response of the carbody under time and frequency domain and the response of the bogie frame in time domain is studied when the vehicle is simulated to run at different speeds. The vehicle ride comfort is evaluated using Sperling Ride index at different simulated speeds. The present analysis suggests that when the vehicle speed is reduced the comfort index increases. This work may be extended to an even higher DoF system. A few more rigid bodies e.g. bolster may be incorporated into the system independently and it may be assigned vertical, lateral and roll DoF. The system may be evaluated for other inputs i.e. sinusoidal, random, cusp etc. The ride comfort may be evaluated based on other standards also.

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