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Autopilot Control System for Damping the Dutch Roll

By

N. C. Ajay Vishwath
Aeronautical Department, Parul Institute of Engineering and Technology, Parul University, India

Mansha kumari B. Patel
Aeronautical Department, Parul Institute of Engineering and Technology, Parul University, India

Swati Chauhan
Aeronautical Department, Parul Institute of Engineering and Technology, Parul University, India

Sujeet Kumar Pandey
Aeronautical Department, Parul Institute of Engineering and Technology, Parul University, India

Abstract

Dutch roll being one of the most difficult disturbing motions of the fixed wing aircraft, could be damped with the help of an autopilot control system. The purpose of this research work is to design such autopilot control system that could damp the oscillations created by this Dutch Roll and thereby bringing the aircraft to equilibrium flight condition. Though conventional engineers use yaw dampers to damp such motions, development of electronics and autopilot board's invention has led Avionics Engineers to design such creation for the benefit of society. Also, the use of Autopilot Control system is reducing the pilot's work especially in UAV industries as well. Most of the mission for small payload capacity carrying mission uses fixed wing UAV and embedding such autopilot boards become conventional in UAV sectors. Though the work done in this manuscript is not for UAV purposes, in future this work would help in adopting the UAV technology for passenger aircrafts as well.

The research work begins with collecting the data of Boeing – 747 and thereby substituting in the lateral directional derivative formula. The stability derivative also requires the stability coefficients. Further these stability derivative and stability coefficient values must be fed to the equation of Dutch Roll Approximation. Using the determinant method, we solve the equations by framing the rudder transfer function and aileron transfer function, through the approximation. These transfer functions are used in a control system where the loop consist of both rate gyro as well as washout circuit which combined to form an autopilot control system. This control system would damp the oscillatory motions produced by Dutch Roll. The control system was solved in Matlab control system tool box by obtaining the root locus and the stability was checked.

Keywords — Autopilot Control System; Dutch Roll;

1. Literature Review

Marcello R. Napolitano implemented the neural controllers with autopilot control laws for a modern high-performance aircraft. He concluded with the two studies, particularly in the first study that deals with on line learning neutral controllers for airspeed and altitude also second study shows the capabilities of the neutral controllers at non-linear conditions. The result shows good performance for the neutral controllers at non-linear conditions with absence of "built-in" robustness capabilities.

Peng Lu aiming for effective way to design and test control flight system for UAVs and he implement the mathematical model by using Mat-lab /Simulink and give validation based on comparison with flight test. The results shows that HIL simulation system could simulate precisely and provide the quality of the system performance work.

Amit Manocha basically investigates on the PID controller for control system with Autopilot, in this he uses genetic algorithms to fed the system input for the Boeing 747-400. Based on that he concluded that genetic algorithms could be used for optimizing the gain values of PID controller also by using genetic algorithms design of a neuro-genetic, fuzzy-genetic or hybrid system could be done for obtaining more improved and prominent autopilot control system.

Dr. Emad N. Abdulwahab discuss about the dynamic instability problem, due to which Dutch roll instability arising. To enhanced the Dutch-roll and lateral

– directional state, the use of yaw damper and washout-filter. For Mig-21 the stability augmentation system (SAS) and numerical model was constructed, which give the numerical result about improved the sideslip angle, roll rate and roll angle for short response time.

Curtis E. Hanson brief about the formation of autopilot on-board the trailing airplane control lateral and vertical spacing under autopilot control production and demonstration of the single and multiple-axis inputs with step commands and frequency sweeps. Even he discusses about control algorithms, navigation and guidance for experimental formation. By this he concludes successful F/A – 18 autopilot system is designed which shows four formation autopilot gain set were tested. The steady-state tracking exceeds the goal of ± 10ft and valve less than ±5ft is shown.

2. Introduction

Lateral Autopilot

In general, if the aircraft is disturbed from its equilibrium due to some gust wind, the aircraft begin to oscillate and will not return back to its initial flight path. So, it becomes pilot's responsibility to align the aircraft along the flight path.

The early lateral autopilots built purposefully to maintain the level of wing and to align the heading path [1].

The autopilot uses two gyros, a vertical gyro for maintaining the wing level and a directional gyro for maintaining the heading path.

These autopilots were not much developed, only few changes were been adopted with which could perform certain changes only. This was accomplished by deflecting rudder and the plane is allowed to yaw thereby overcoming the disturbances.

But these autopilots won't fly modern fixed wing aircrafts, as modern aircrafts have high maneuvering capability and high-performance capabilities, also these old autopilots lack in maneuverability and posses light damping nature especially during Dutch Roll oscillations.

Hence, Artificial dampers are being installed in the modern fixed wing aircrafts. To align the heading flight path, extra maneuverability is added by achieving turn control, by aileron deflection through sending proper signals to the rudder.

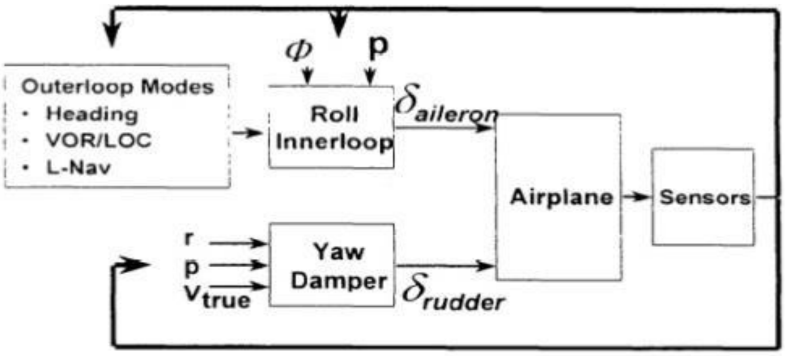


Fig.1. *Block diagram of Lateral autopilot*

Dutch Roll [2]

Dutch roll is a disturbing motion that produces yaw coupled roll. The oscillation produced are of same frequency but they are out of phase with each other. These disturbances would last only for 3 to 15 seconds, but still, it annoys the pilots as well as the passengers present inside in the aircraft.

Though in most light fixed wing aircrafts, the Dutch Rolls are being well damped, the damping would get degraded if either velocity increases or altitude increases.

And in general, these Dutch Roll gets stabilized through the installation of yaw damper.

Fig2. *Dutch Roll Oscillations in fixed wing aircraft*

3. Methods to Damp the Dutch Roll:

In place of p and r, the ϕ and ψ will be substituted for framing the lateral equations of motion. This could be possibly done if assumptions of small perturbations were made from equilibrium.

Ψ = angular velocity about vertical of fixed wing aircraft
r= measured w.r.t Z axis of fixed wing aircraft
Ψ and r will be considered equal for smaller angles, but not in case of larger angle.

If we consider lateral autopilots, especially for correcting flight path and obtaining the coordination, smaller roll angle compulsion is not there, so usage r in place of ψ will be more appropriate while considering angular velocities about the Z-axis of the fixed wing aircraft.

These considerations will make yaw rate gyro take r as the angular velocity in place of ψ. Until θ remains zero,

Identicality of the angular velocities p and ϕ about X-axis of the aircraft are maintained. In usual case, the Dutch Roll mode would get excited because of the rudder. Yawing moment produced due to the deflection of aileron produces the sideslip response with the Dutch Roll along with the yaw rate from deflection of aileron.

Because of the above stated reasons, the oscillations produced by the Dutch Roll gets damped in a way such that the yaw rate would get detected by a rate gyro and signal produced by this would be used for deflecting rudder.

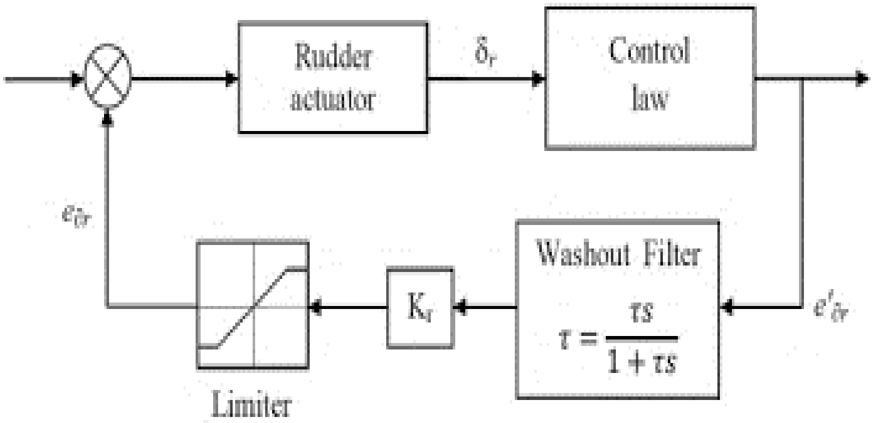


Fig3. *Dutch Roll Damper system with washer circuit installed* [3]

The block diagram given above is the Dutch Roll Damper along with the washout circuit installed. The output is generated by this during the transient period only. In the steady state, if the yaw rate signal doesn't become zero, the output from the yaw rate gyro would produce a positive deflection of rudder for a positive yaw rate. But it will get terminated in uncoordinated maneuvering.

Washer Circuit Transfer Function is $\tau s/(1+\tau s)$

In order to bring down the root locus for the given time constant values, we need to demonstrate the effect produced by changing the time constant value in the washout circuit.

The block diagram is reduced by simplifying the transfer function and the blocks are modified. As the root locus used here is for rudder input alone, because of this, we omit the input of aileron.

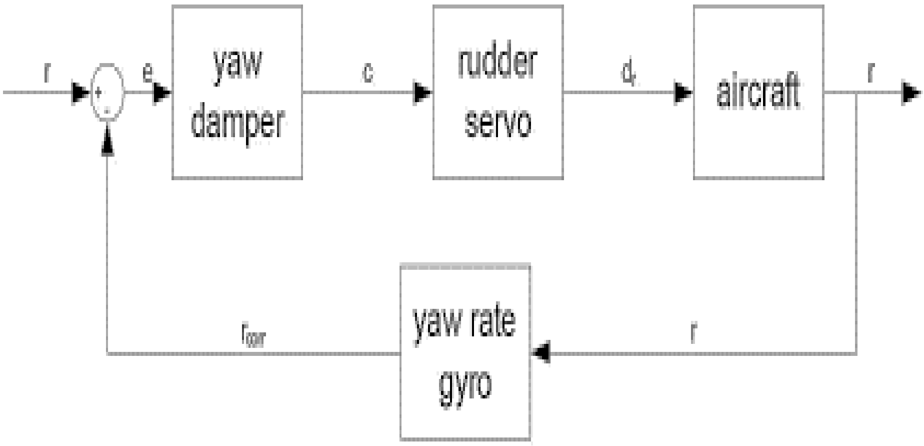


Fig 4: *Typical control system with yaw damper & yaw rate gyro*

4. Methodology

Boeing 747 Specifications [4]:
Weight =636600lb
C.G@ 25% MAC
I_x=18.2*102 slug-ft^2
I_y=33.1*106 slug-ft^2
I_z=49.7*106 slug-ft^2
I_{xz}=0.97*106 slug-ft^2
Chord c= 27.3ft
Wing span b=195.68ft
Wing planform Area S=5500ft^2

Table 1: *Lateral Derivative Coefficients for Boeing-747 at M=0.25, sea level condition*

S.no	Lateral Derivative Coefficients for Boeing-747	Symbol	Value
1.	Side Force Coefficient with respect to sideslip angle	$C_{y\beta}$	-0.96
2.	Rolling Moment Coefficient with respect to sideslip angle	$C_{l\beta}$	-0.221
3.	Yawing Moment Coefficient with respect to sideslip angle	$C_{n\beta}$	0.150
4.	Rolling Moment Coefficient with respect to rolling angle	C_{lp}	-0.45
5.	Yawing Moment Coefficient with respect to rolling angle	C_{np}	-0.121
6.	Rolling Moment Coefficient with respect to yawing angle	C_{lr}	0.101
7.	Yawing Moment Coefficient with respect to yawing angle	C_{nr}	-0.30
8.	Rolling Moment Coefficient with respect to aileron deflection angle	$C_{l_{\delta a}}$	0.0461
9.	Yawing Moment Coefficient with respect to aileron deflection angle	$C_{n_{\delta a}}$	0.0064
10.	Side Force Coefficient with respect to rudder deflection angle	$C_{y_{\delta r}}$	0.175
11.	Rolling Moment Coefficient with respect to rudder deflection angle	$C_{l_{\delta r}}$	0.007
12.	Yawing Moment Coefficient with respect to rudder deflection angle	$C_{n_{\delta r}}$	-0.109
13.	Side Force Coefficient with respect to rolling angle	C_{yp}	0
14.	Side Force Coefficient with respect to yawing angle	C_{yr}	0
15.	Side Force Coefficient with respect to aileron deflection angle	$C_{y_{\delta a}}$	0

Lateral Directional Derivatives

Side Force due to Sideslip	$Y_\beta = \frac{QSC_{y\beta}}{m}$ (ft/s^2)	(1)
Side Force due to Roll	$Y_p = \frac{QSbC_{yp}}{2mu_o}$ (ft/s)	(2)
Side Force due to Yaw	$Y_r = \frac{QSbC_{yr}}{2mu_o}$ (ft/s)	(3)
Side Force due to Aileron Deflection	$Y_{\delta a} = \frac{QSC_{y\delta a}}{m}$ (ft/s^2)	(4)
Side Force due to Rudder Deflection	$Y_{\delta r} = \frac{QSC_{y\delta r}}{m}$ (ft/s^2)	(5)
Yawing Moment due to Sideslip	$N_\beta = \frac{QSbC_{n\beta}}{\mathrm{Iz}}$ (s^{-2})	(6)
Yawing Moment due to Roll	$N_p = \frac{QSb^2C_{np}}{2\mathrm{Ix}u_o}$ (s^{-1})	(7)
Yawing Moment due to Yaw	$N_r = \frac{QSb^2C_{nr}}{2\mathrm{Ix}u_o}$ (s^{-1})	(8)
Yawing Moment due to Aileron Deflection	$N_{\delta a} = \frac{QSbC_{n\delta a}}{\mathrm{Iz}}$ (s^{-2})	(9)
Yawing Moment due to Rudder Deflection	$N_{\delta r} = \frac{QSbC_{n\delta r}}{\mathrm{Iz}}$ (s^{-2})	(10)
Rolling Moment due to Sideslip	$L_\beta = \frac{QSbC_{l\beta}}{\mathrm{Ix}}$ (s^{-2})	(11)
Rolling Moment due to Roll	$L_p = \frac{QSb^2C_{lp}}{2\mathrm{Ix}u_o}$ (s^{-1})	(12)
Rolling Moment due to Yaw	$L_r = \frac{QSb^2C_{lr}}{2\mathrm{Ix}u_o}$ (s^{-1})	(13)
Rolling Moment due to Aileron Deflection	$L_{\delta a} = \frac{QSbC_{l\delta a}}{\mathrm{Iz}}$ (s^{-2})	(14)
Rolling Moment due to Rudder Deflection	$L_{\delta r} = \frac{QSbC_{l\delta r}}{\mathrm{Iz}}$ (s^{-2})	(15)

Where,

Iy	Pitching Moment of Inertia
Ix	Rolling Moment of Inertia
Iz	Yawing Moment of Inertia
Ixz	Product of Inertia about x-z Axis
u_o	Reference Flight Speed
Q	Dynamic Pressure
c	Mean Chord
b	Wing Span
S	Wing Planform Area

Dutch Roll Approximation:

The approximate equations can be shown to be

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{u_o} & -\left(1 - \frac{Y_r}{u_o}\right) \\ N_\beta & N_r \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix} + \begin{bmatrix} \frac{Y_{\delta r}}{u_o} & 0 \\ N_{\delta r} & N_{\delta a} \end{bmatrix} \begin{bmatrix} \Delta\delta r \\ \Delta\delta a \end{bmatrix} \tag{16}$$

Laplace transformation operation and rearranging the terms yield to

$$\left(s - \frac{Y_\beta}{u_o}\right) \Delta\beta(s) + \left(1 - \frac{Y_r}{u_o}\right) \Delta r(s) = \frac{Y_{\delta r}}{u_o} \Delta\delta r(s) \tag{17}$$

$$-N_\beta \Delta\beta(s) + (s - N_r)\, \Delta r(s) = N_{\delta a}\, \Delta\delta a(s) + N_{\delta r}\, \Delta\delta r(s) \tag{18}$$

Transfer Functions $\frac{\Delta\beta(s)}{\Delta\delta r(s)}$, $\frac{\Delta r(s)}{\Delta\delta r(s)}$, $\frac{\Delta\beta(s)}{\Delta\delta a(s)}$ and $\frac{\Delta r(s)}{\Delta\delta a(s)}$ could be found out by making $\Delta\delta a(s)$ to zero and thereby

Solving the functions $\frac{\Delta\beta(s)}{\Delta\delta r(s)}$ and $\frac{\Delta r(s)}{\Delta\delta r(s)}$.

Put $\Delta\delta r(s)$ equals to zero and thereby

Solving the functions $\frac{\Delta\beta(s)}{\Delta\delta a(s)}$ and $\frac{\Delta r(s)}{\Delta\delta a(s)}$.

Transfer Functions $\frac{\Delta\beta(s)}{\Delta\delta r(s)}$ and $\frac{\Delta r(s)}{\Delta\delta r(s)}$ found out are as follows:

$$(s-\frac{Y_\beta}{u_o})\frac{\Delta\beta(s)}{\Delta\delta r(s)}+(1-\frac{Y_r}{u_o})\frac{\Delta r(s)}{\Delta\delta r(s)}=\frac{Y_{\delta r}}{u_o} \tag{19}$$

$$-N_\beta\frac{\Delta\beta(s)}{\Delta\delta r(s)}+(s-N_r)\frac{\Delta r(s)}{\Delta\delta r(s)}=N_{\delta r} \tag{20}$$

$$\frac{\Delta\beta(s)}{\Delta\delta r(s)}=\frac{\begin{vmatrix}\frac{Y_{\delta r}}{u_o} & 1-\frac{Y_r}{u_o}\\ N_{\delta r} & s-N_r\end{vmatrix}}{\begin{vmatrix}s-\frac{Y_\beta}{u_o} & 1-\frac{Y_r}{u_o}\\ -N_\beta & s-N_r\end{vmatrix}} \quad \text{and,} \quad \frac{\Delta r(s)}{\Delta\delta r(s)}=\frac{\begin{vmatrix}s-\frac{Y_\beta}{u_o} & \frac{Y_{\delta r}}{u_o}\\ -N_\beta & N_{\delta r}\end{vmatrix}}{\begin{vmatrix}s-\frac{Y_\beta}{u_o} & 1-\frac{Y_r}{u_o}\\ -N_\beta & s-N_r\end{vmatrix}}$$

$$\frac{\Delta\beta(s)}{\Delta\delta r(s)}=\frac{N^\beta_{\delta r}(s)}{\Delta DR(s)}=\frac{A_\beta s+B_\beta}{As^2+Bs+C} \tag{21}$$

$$\frac{\Delta r(s)}{\Delta\delta r(s)}=\frac{N^r_{\delta r}(s)}{\Delta DR(s)}=\frac{A_r s+B_r}{As^2+Bs+C} \tag{22}$$

Similarly, the aileron transfer function can be shown to be

$$\frac{\Delta\beta(s)}{\Delta\delta r(s)}=\frac{N^\beta_{\delta r}(s)}{\Delta DR(s)}=\frac{A_\beta s+B_\beta}{As^2+Bs+C} \tag{23}$$

$$\frac{\Delta r(s)}{\Delta\delta a(s)}=\frac{N^r_{\delta a}(s)}{\Delta DR(s)}=\frac{A_r s+B_r}{As^2+Bs+C} \tag{24}$$

Dutch Roll transfer function approximations

$$\Delta DR(s)\rightarrow \text{A=1, B=}\frac{-(Y_\beta+u_o N_r)}{u_o}\text{, C=}\frac{(Y_\beta N_r-N_\beta Y_r+N_\beta u_o)}{u_o}$$

$$N^\beta_{\delta r}(s)\rightarrow A_\beta=\frac{Y_r}{u_o},\ B_\beta=\frac{(Y_r N_{\delta r}-Y_{\delta r}N_r-N_{\delta r}u_o)}{u_o}$$

$$N^r_{\delta r}(s)\rightarrow A_r=N_{\delta r},B_r=\frac{(N_\beta Y_{\delta r}-Y_\beta N_{\delta r})}{u_o}$$

$$N^\beta_{\delta a}(s)\rightarrow A_\beta=0,\ B_\beta=\frac{(Y_r N_{\delta a}-N_{\delta a}u_o)}{u_o}$$

$$N^r_{\delta a}(s)\rightarrow A_r=N_{\delta a},B_r=\frac{-Y_\beta N_{\delta a}}{u_o}$$

5. Mathematical Calculations

For sea level conditions

Mach number, M=0.25
Velocity of sound, a= 1125.33ft/sec
Mach no. M=$\frac{u_o}{a}$
So, the Velocity, u_o = 281.33ft/sec
Density, ρ_o = 0.00237slug/ft^3
Dynamic velocity, Q=0.5xρ_ox$u_o{}^2$ = 94.18slug/ft/sec^2
Mass, m = $\frac{W}{g}$ = 19770.19 slugs

Directional Derivatives which are found by using Lateral Derivative Coefficients:

$$Y_\beta = \frac{QSC_{y\beta}}{m} = -25.15(\text{ft/s}^2)$$

$$N_\beta = \frac{QSbC_{n\beta}}{\text{Iz}} = 0.31\ (\text{s}^{-2})$$

$$L_\beta = \frac{QSbC_{l\beta}}{\text{Ix}} = -1.23\ (\text{s}^{-2})$$

$$Y_p = \frac{QSbC_{yp}}{2mu_o} = 0(\text{ft/s})$$

$$N_p = \frac{QSb^2C_{np}}{2\text{Ix}u_o} = -0.23\ (\text{s}^{-1})$$

$$L_p = \frac{QSb^2C_{lp}}{2\text{Ix}u_o} = -0.875(\text{s}^{-1})$$

$$Y_r = \frac{QSbC_{yr}}{2mu_o} = 0\ (\text{ft/s})$$

$$N_r = \frac{QSb^2C_{nr}}{2\text{Ix}u_o} = -0.58\ (\text{s}^{-1})$$

$$L_r = \frac{QSb^2C_{lr}}{2\text{Ix}u_o} = 0.2(\text{s}^{-1})$$

$$Y_{\delta a} = \frac{QSC_{y_{\delta a}}}{m} = 0\ (\text{ft/s}^2)$$

$$N_{\delta a} = \frac{QSbC_{n_{\delta a}}}{\text{Iz}} = 0.01\ (\text{s}^{-2})$$

$$L_{\delta a} = \frac{QSbC_{l_{\delta a}}}{\text{Iz}} = 0.26(\text{s}^{-2})$$

$$Y_{\delta r} = \frac{QSC_{y_{\delta r}}}{m} = 4.59\ (\text{ft/s}^2)$$

$$N_{\delta r} = \frac{QSbC_{n_{\delta r}}}{\text{Iz}} = -0.22\ (\text{s}^{-2})$$

$$L_{\delta r}=\frac{QSbC_{l_{\delta r}}}{\mathrm{Iz}}=0.04(\mathrm{s}^{-2})$$

Table 2: *Values of Lateral derivative coefficients and Directional derivatives*

S.no	Lateral derivative coefficients	Lateral derivative coefficients values	Directional derivatives	Directional derivatives values
1.	$C_{y\beta}$	-0.96	Y_β	-25.15
2.	$C_{l\beta}$	-0.221	L_β	-1.23
3.	$C_{n\beta}$	0.150	N_β	0.31
4.	C_{lp}	-0.45	L_p	-0.875
5.	C_{np}	-0.121	N_p	-0.23
6.	C_{lr}	0.101	L_r	0.2
7.	C_{nr}	-0.30	N_r	-0.58
8.	$C_{l_{\delta a}}$	0.0461	$L_{\delta a}$	0.26
9.	$C_{n_{\delta a}}$	0.0064	$N_{\delta a}$	0.01
10.	$C_{y_{\delta r}}$	0.175	$Y_{\delta r}$	4.59
11.	$C_{l_{\delta r}}$	0.007	$L_{\delta r}$	0.04
12.	$C_{n_{\delta r}}$	-0.109	$N_{\delta r}$	-0.22
13.	C_{yp}	0	Y_p	0
14.	C_{yr}	0	Y_r	0
15.	$C_{y_{\delta a}}$	0	$Y_{\delta a}$	0

Transfer Functions that are found out by solving the the directional derivative equations:

$$\frac{\Delta\beta(s)}{\Delta\delta r(s)}=\frac{N^{\beta}_{\delta r}(s)}{\Delta DR(s)}=\frac{A_\beta s+B_\beta}{As^2+Bs+C}$$

$$\frac{\Delta\beta(s)}{\Delta\delta r(s)}=\frac{64.5548}{281.33s^2+188.32s+101.8} \tag{25}$$

$$\frac{\Delta r(s)}{\Delta\delta r(s)}=\frac{-61.89s-4.11}{281.33s^2+188.32s+101.8} \tag{26}$$

$$\frac{\Delta\beta(s)}{\Delta\delta a(s)}=\frac{-2.81}{281.33s^2+188.32s+101.8} \tag{27}$$

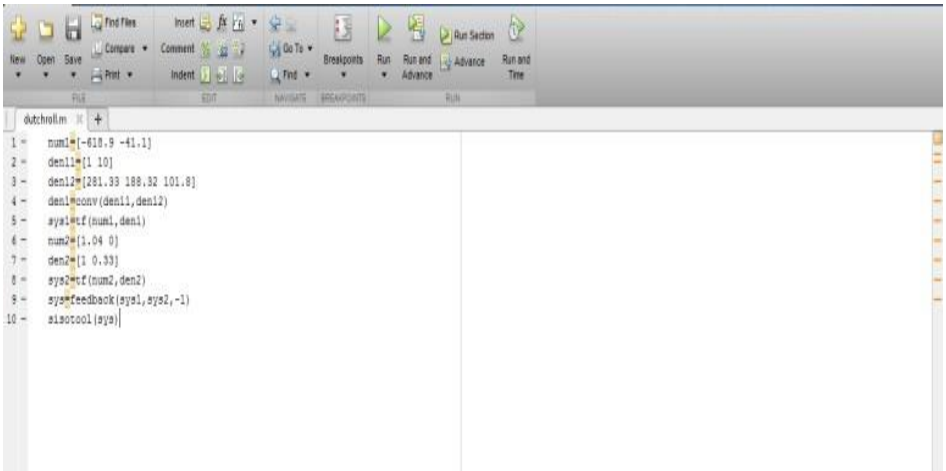
$$\frac{\Delta r(s)}{\Delta\delta a(s)}=\frac{2.81s+0.25}{281.33s^2+188.32s+101.8} \tag{28}$$

6. Results

If we feed transfer function of the rudder in the fixed wing aircraft transfer function block of control system in the Dutch Roll Damper of the control system and giving rudder servo equation [6] as 10/(s+10)

Equation of Washout Circuit is s/(s+1/τ).
S(yrg) is given as 1.04V/(deg/sec)

Once the final equation is solved through MATLAB, we use SISO toolbar to obtain the individual root locus solution and the stability of the system could be verified through the plot of graph obtained by changing the gain values.



```
num1=[-618.9 -41.1]
den11=[1 10]
den12=[281.33 188.32 101.8]
den1=conv(den11,den12)
sys1=tf(num1,den1)
num2=[1.04 0]
den2=[1 0.33]
sys2=tf(num2,den2)
sys=feedback(sys1,sys2,-1)
sisotool(sys)
```

Fig5. *Code for solving the Transfer Function in MATLAB Software*

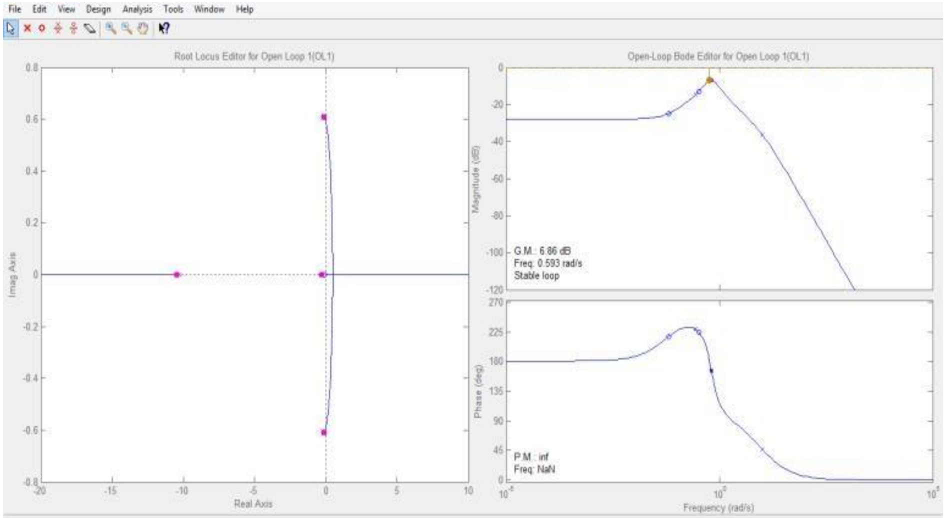


Fig6. *Plot of the solution for Transfer Function in MATLAB Software*

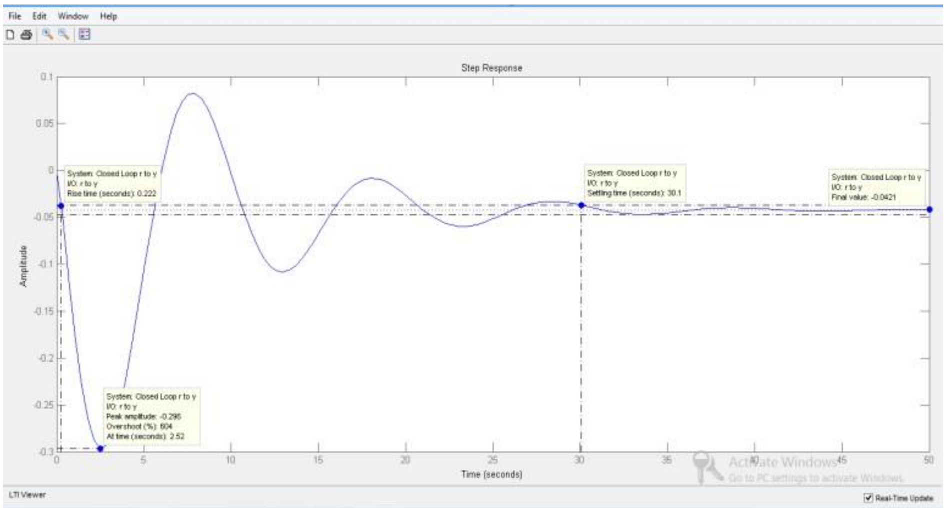


Fig 7. *Step Response when gain k=1, damping ratio=0.205, natural frequency=0.623, stable loop* [8]

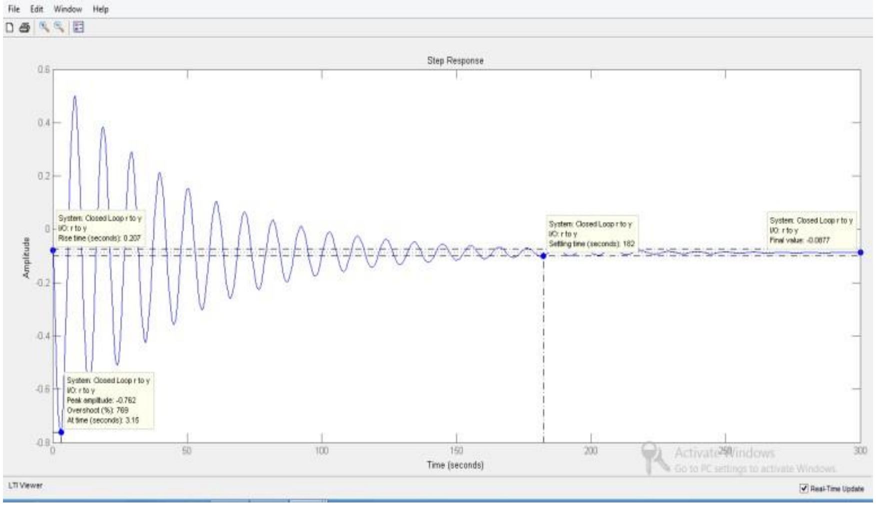


Fig 8. *Step Response when gain k=2, damping ratio=0.036, natural frequency=0.598, stable loop*

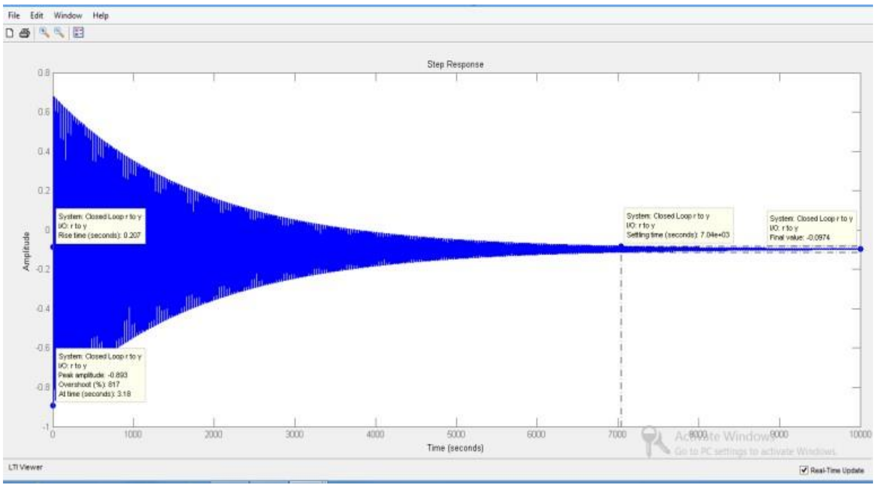


Fig 9. *Step Response when gain k=2.2, damping ratio=0, natural frequency=0.594, stable loop*

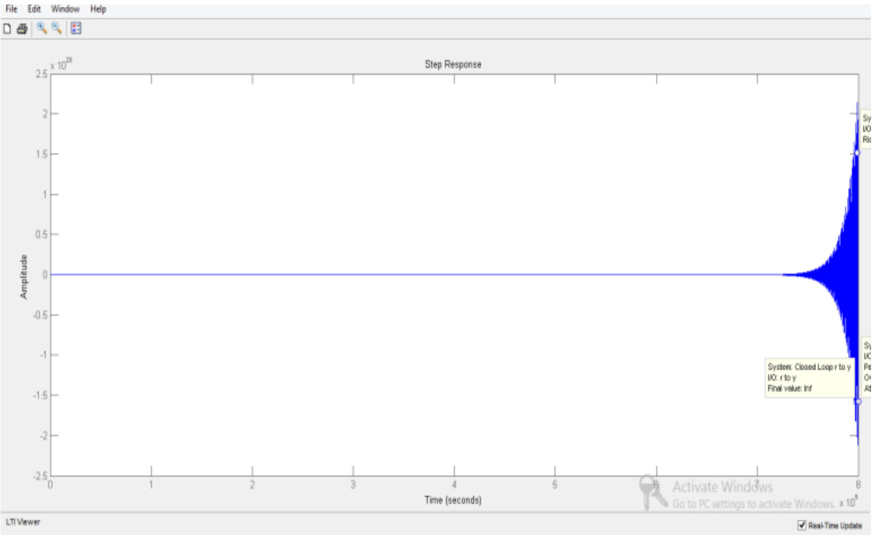


Fig 10. *Step Response when gain k=2.2, damping ratio=-0, natural frequency=0.593, unstable loop*

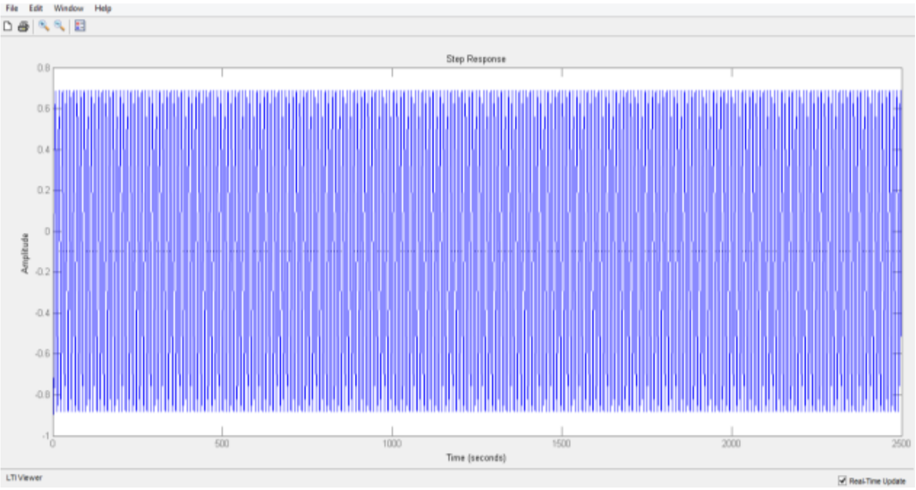


Fig 11. *Step Response when gain k=2.203441, damping ratio=-0, natural frequency=0.593, neutrally stable loop*

7. Conclusions and Recommendations

a) Autopilot system that requires to damp the oscillation produced Dutch Roll motion is designed successfully.

b) The Transfer Function obtained through mathematical calculation is working perfectly and also after putting it to the Autopilot Control System, the Control System of the aircraft is Stable [9].

c) In case the designed Autopilot Control System is formed into the hardware system and deployed in

Boeing-747 aircraft, the aeroplane will become stable to its flight path automatically.

d) The Autopilot Control System is stable for gain values of 0.000576 to 2.2.

Acknowledgements

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