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An Application Of The Extreme Value Theory Using Q-Q Plot Representation In Financial Engineering: The Estimation Of The Var In The Moroccan Stock Market During The Occurrence Of Some Rare Past Events.

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Abstract

Since the year 2003, Morocco has seen some rare events, which are considered as unlikely risks in term of financial engineering and risk management. They had effects on the national economy. The objective of this article is to apply the theory of extreme values in the estimation of the value at risk in the Moroccan financial market and specifically in the stock market during the period of occurrence of these events on the one side, and on the other side to compare the estimate by this method with the real losses. This estimation of the value at risk, which represents the maximum loss that should not be exceeded for a given probability in a given horizon, will be calculated during the occurrence of four rare events: The Casablanca bombings on 16 May 2003,The banking and financial crisis of 2007-2008,The pandemic of covid-19 on 2019-2020,The Russian-Ukrainian conflict on 2022.The estimate is going to be applied to the MASI index which is the principal stock market index of the Moroccan Stock Exchange. It is constituted of all the stocks listed on the Casablanca Stock Exchange.

Keywords: Unlikely risks, VaR, extreme value theory, financial engineering, rare events.

Introduction

Among the rare events that have been produced in Morocco we can mention: The Casablanca bombings on 16 May 2003, the banking and financial crisis of 2007-2008, the pandemic of covid-19 on 2019-2020 and the Russian-Ukrainian conflict on 2022.

The attacks in Casablanca were a group of suicidal bombings that happened on May 16, 2003 in the Moroccan city of Casablanca. They were executed by a group of terrorists. The result was severe.

The 2008 financial crisis is a serious economic crisis that occurred at the beginning of the twenty-first century. It is the most serious financial crisis since the 1929 depression.

The economic crisis of the Covid-19 pandemic is a world' economic crisis, caused by the Covid-19 pandemic and the isolation of a large number of countries.

The Russian-Ukrainian conflict is a diplomatic and military conflict between Russia and the Ukraine. It began in 24 February 2022.

These rare events had clear impacts on the Moroccan economy. The Moroccan stock market illustrates these impacts.

In statistics these rare events are considered extreme values. The extreme value theory allows us to manage these values. This theory gives the comportment of the maximum of values taken by the values of variables. It is used in many fields including finance.

Literature review and hypothesis development:

The concept of value-at-risk:

The value at risk can be expressed as the maximum anticipated loss of a portfolio on a given time horizon and at a certain confidence level.

The variables that are the basis of the VaR are:

The probability that the loss will not be higher than expected.

The forecast horizon: it's the time frame of the estimated VaR. It is assumed that within this forecast horizon, the portfolio does not change.

Return volatility.

Consider $r_1, r_2, \ldots, r_n$ as random variables expressing financial returns of a given asset.

Consider $F(r) = \Pr(r_t < r / \Omega_{t-1})$ as the cumulative distribution function of the set of information Ω_{t-1} available at time t-1.

Suppose that r_t follows a stochastic process:

$$r_t = \mu_t + \varepsilon_t = \mu_t + \sigma_t z_t \quad (1)$$

With $\mu_t = E(\varepsilon_t / \Omega_{t-1})$, $\sigma_t^2 = E(\varepsilon_t^2 / \Omega_{t-1})$ et $z_t = \varepsilon_t / \sigma_t$ admits a conditional distribution function G(z), $G(z) = \Pr(z_t < z_t / \Omega_{t-1})$.

The VaR(α) is written as the quantile of the probability distribution of financial returns:

$$F(VaR(\alpha)) = \Pr(r_t < VaR(\alpha)) = \alpha \quad (2)$$

This quantile can be estimated in two different methods:

To invert the distribution function of financial returns F(r).

To invert the distribution function of financial returns standardized G(z).

If the second method is chosen, it will be necessary to estimate μ_t and σ^2_t

$$VaR(\alpha) = F^{-1}(\alpha) = \mu_t + \sigma_t G^{-1}(\alpha) \quad (3)$$

Therefore, the estimation of VaR involves the specification of either F(r),G(z) and μ_t and σ^2_t.[18]

The concept of the theory of extreme values:

Extreme Value Theory is focused on the properties of distribution tails. It can be used to assess the risk of occurrence of extreme events over a given level. These extreme events are often seen as aberrations but they present a real risk. This theory permits to improve considerably the calculation of the VaR for very high levels of the high values of the parameter α. [2]

Consider X as a random variable whose distribution function F () is unknown:

$$F(x) = \Pr(X \leq x) \quad (4)$$

Consider μ as a value of X located in the right tail of the distribution. We can consider it as an "extreme value". We would like to have information on the probability that X exceeds μ.[11]bThe probability that the difference between X and μ does not exceed an amount y, in the case where X would exceed the value μ, is donating by :

$$F_\mu(y) = \Pr(X - \mu \leq y \;/\; X > \mu) = \frac{F(y+\mu) - F(\mu)}{1 - F(\mu)} \quad (5)$$

$F(y + \mu) - F(\mu)$ is the probability that X is between μ and μ+y and $1 - F(\mu)$ is the probability for X to exceed μ.[10]

We conclude, conversely, that the probability that the difference between X and μ exceeds y, knowing that X already exceeds u, is given by 1 - $F_\mu(y)$. Knowing that X > μ. [11]

The Balkema-de Haan-Pickands (BdHP) theorem (Balkema and de Haan, 1974; Pickands,1975) says that, even if the distribution of X is a priori unknown, it is still possible to approach this unknown distribution validly, but only in its tail of distribution, bay generalized Pareto law whose parameters (ξ,β) must be estimated. This means that the distribution function of excess returns y over μ,$F_\mu(y)$, canbe approximated by $G_{\xi,\beta}(y)$ the distribution function of the generalized Pareto distribution with parameters ξ and β defined as

$$F_\mu(y) = G_{\xi,\beta}(y) = 1 - \left(1 + \xi \frac{y}{\beta}\right)^{-\frac{1}{\xi}}, \text{ if } \xi \neq 0 \quad (6)$$

ξ is the form parameter: it gives information about the more or less thick character of the tail distribution. The more ξ is high, the more the tail of the distribution the thicker. Therefore, the probability of having extreme returns is high. In contrast, when ξ tends to zero, the tail of the distribution tends to have the same properties as those of a normal distribution and the probability of extreme returns is low. The β parameter is only a levelling parameter. This pair of parameters, ξ and β of the generalized Pareto distribution are a priori unknowns; however, they can be estimated for a pre-chosen μ value such that the theorem applies. The levelμ, should not be too high or too low. We can estimate the values of ξ and β by the likelihood method. [2]

Application of the extreme value theory for the calculation of the var :

The objective is to approximate F(x) for values of X that are high enough. We can write

$$F(x) = F(\mu + y) = F_\mu(y).(1 + F(\mu)) + F(\mu) = G_{\xi,\beta}(x - \mu).(1 - F(\mu)) + F(\mu)) \quad (7)$$

we can approximate $F_\mu(y)$ by $G_{\xi,\beta}(x - \mu)$.Consider n as the total number of observations. [2]

in the portfolio returns data and n_μ is the number of observations where X > μ.the estimator of F(μ) is $\hat{F}(\mu) = (n - n_\mu).\, n^{-1}$, It is the proportion of X that are greater than μ among the n observations that precede the date at which the VaR is estimated. The estimate of 1-F(μ) is therefore equal to $\frac{n_\mu}{n}$. By substituting the different parameters of the generalized Pareto distribution by their estimates, we find for X>μ that [2]

$$\hat{F}(x) = \hat{F}(\mu + y) = \frac{n-n_\mu}{n} + G_{\hat{\xi},\hat{\beta}}(x-\mu)\cdot\left(\frac{n_\mu}{n}\right) \quad (8)$$

Therefore:

$$\hat{F}(x) = \hat{F}(\mu + y) = 1 - \frac{n_\mu}{n}\cdot\left(1 + \hat{\xi}\,\frac{x-\mu}{\hat{\beta}}\right)^{-\frac{1}{\hat{\xi}}} \quad (9)$$

So:

$$P(x>\mu) = 1 - \hat{F}(\mu + y) = \frac{n_\mu}{n}\left(1 - G_{\hat{\xi},\hat{\beta}}(x-\mu)\right) = \frac{n_\mu}{n}\cdot\left(1 + \hat{\xi}\cdot\frac{x-\mu}{\hat{\beta}}\right)^{-\frac{1}{\hat{\xi}}} \quad (10)$$

The VaR at the level α (with $\alpha > F(\mu)$) is calculated as the particular value of $x = \mu + y$ for which we verify that:

$$\hat{F}(\mu + y) = \alpha \quad (11)$$

The estimated VaR of level x is therefore such that: [12]

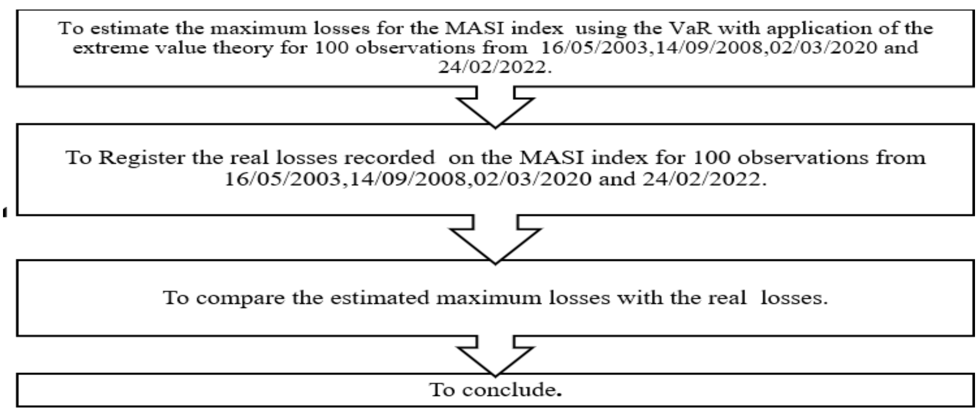
$$\widehat{VaR}_\alpha = \mu + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{n_\mu}(1-\alpha)\right)^{-\hat{\xi}} - 1\right),\ \text{with } \alpha > 1 - \frac{n_\mu}{n} \quad (12)$$

Developing hypothesis:

For the four rare events presented in this paper, we test the null hypothesis H0: The application of the extreme value theorem using Q-Q plot representation in the estimation of the value at risk is useful in the Moroccan stock market, versus the alternative hypothesis H1: the application of the extreme value theorem using Q-Q plot representation in the estimation of the value at risk is not useful in the Moroccan stock market.

Research methodology:

The objective of this work is to apply the extreme value theory to the estimation of the value at risk for four rare events that had effects on the Moroccan economy between 2003 and 2022. The selected events are: The Casablanca bombings on 16 May 2003, The banking and financial crisis of 2007-2008, The pandemic of covid-19 on 2019-2020, The Russian-Ukrainian conflict on 2022. This four events are examples of unlikely risks. The first is a terrorist risk. The second is an economic risk. The third is a sanitary risk. The last one is a geopolitical risk. This estimate will be applied to a time period of the MASI index during the period 2003 to 2022. In this estimation we will calculate the maximum loss that should not be exceeded for 100 observations from the dates of occurrence of the mentioned events for a confidence level of 99% by the VaR model with application of the extreme value theory. The approach can be summarized as follows:



Descriptive statistics, Results, and discussion

Descriptive statistics:

As we have already explained, to apply the extreme value theory to VaR we must first estimate ξ and β.To do this we need to have the value of μ then apply the maximum likelihood method.

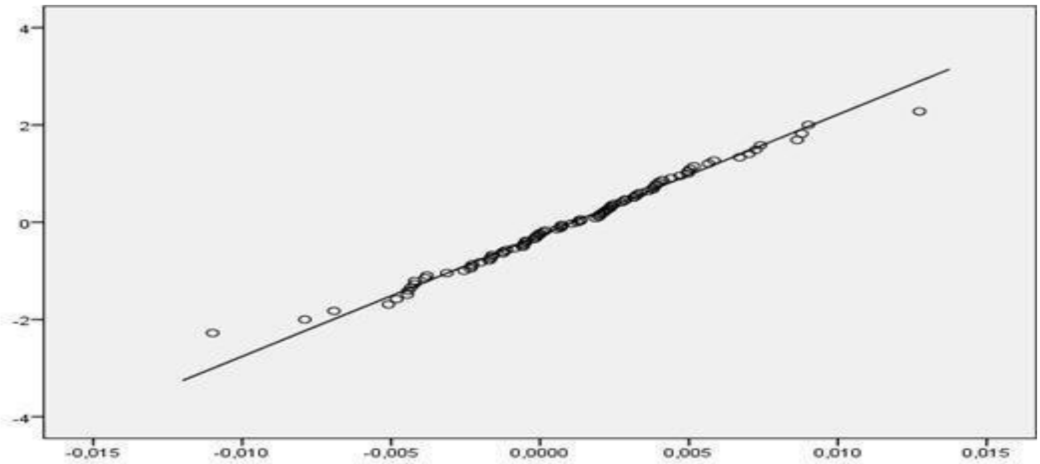
In theory, it is possible to choose the value μ by optimizing the trade-off between bias and efficiency. In effector r a low value μ, the estimators will be biased and for a high value μ, the number of extreme observations considered in the study is reduced, and consequently the variance is overestimated.

There are effective techniques for estimating the value μ: the Quantile-Quantile plot, the Hill estimator and the excess mean function. In our case we use the Quantile-Quantile representation (Q-Q plot).

Suppose that $X_1, \dots, X_n$ are independent observations, with distribution function F with $X_1 \geq X_2 \geq \cdots \geq X_n$. The quantile-quantile graph is defined by:

$$X_K, F\left(\frac{n-k+1}{n+1}\right), k = 1, \dots, n.$$

The pertinence of the choice of the value μ can be verified visually by analyzing the quantile-quantile representation. It is the value from which we notice a certain linearity of the observations.



<Fig. 1> The Quantile-Quantile representation of the sample studied on May 16, 2003.

After visually reading the value of μ , we can proceed to the application of the maximum likelihood method to estimate ξ and $\hat{\beta}$ This method consists in looking for ξ and $\hat{\beta}$ allowing the maximization of the following expression:

$$\sum_{i=1}^{n_\mu}\left[\ln \frac{1}{\beta}\left(1 + \frac{\xi(r_i - \mu)}{\beta}\right)^{-1/\xi - 1}\right]$$

The table 1 shows the results of our optimization applied on the above mentioned sample of May 16, 2003. The table 2 shows the results of our optimization applied on the above mentioned sample of September 14, 2008. The table 3 shows the results of our optimization applied on the above mentioned sample of Mars 02, 2020.

Results and discussion:

For May 16, 2003 the results of the optimization are as follows ($V_P$is the portfolio's value):

<**Table 1**> The results of the optimization on May 16, 2003.

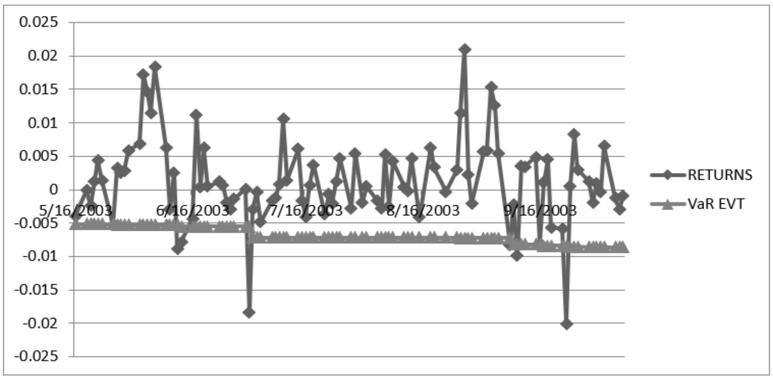
$\vert \mu \vert$	0.00509 V_P
β	0.0000001
ξ	0.09 V_P
N	89
$\mathbf{n}_\mu$	5
Confidence level	99%

Source: our treatment methods.

The calculated value at risk for that day is therefore

$$\hat{VaR}_{99\%} = 0.00509. V_P + \frac{0,0000001}{0,09} . V_P . \left(\left(\frac{89}{5}(1 - 0,99) \right)^{-0,09} - 1 \right) \cong 0.0051. V_P$$

In principle, if we test a 99% VaR, we should expect the daily portfolio returns to exceed the VaR one time in 100 observations. For this we calculate the VaR for the 100 days which follow the shock. This means the period between May 16, 2003 and October 09, 2003.The results are shown in figure 2.



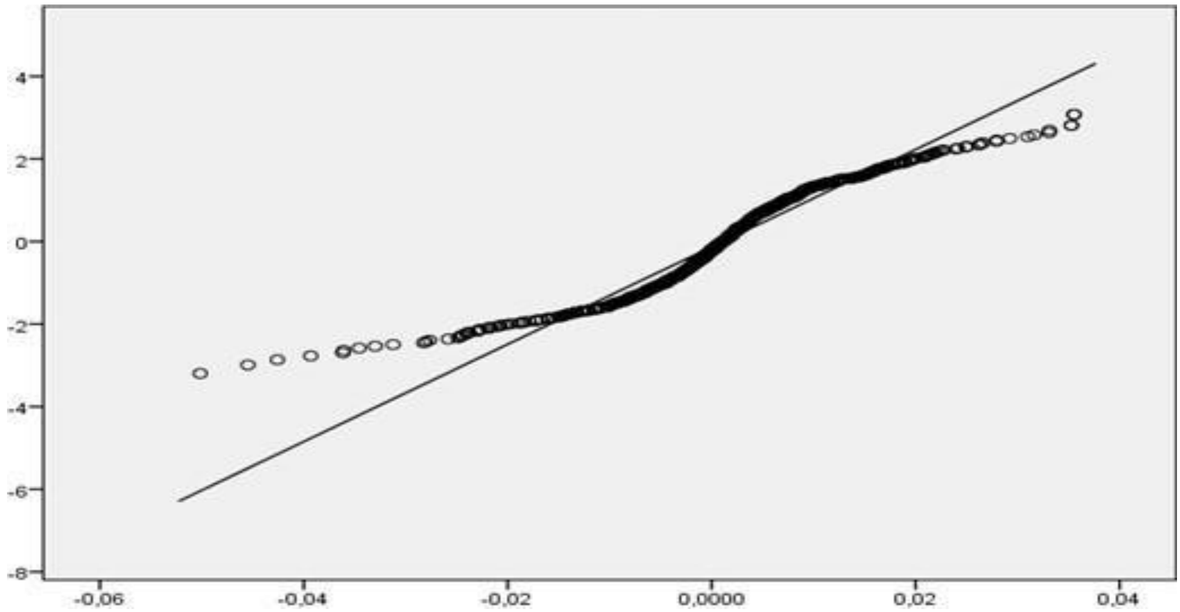
<**Fig. 2**> Comparison between real losses and estimated losses for for the 100 observations after May 16, 2003.

Similarly, the table 2 and the figure 4 present the results obtained for September 14, 2008.

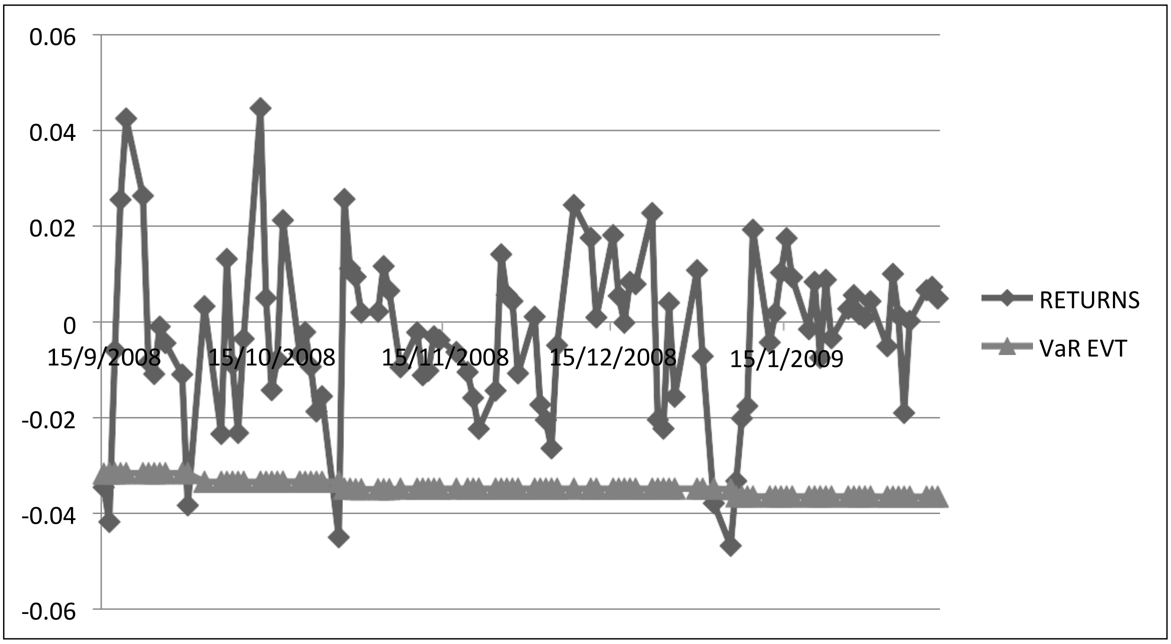
<**Table 2**> The results of the optimization on September 14, 2008**.**

$\vert \mu \vert$	0.03125 V_P
β	0.0000001
ξ	0.09 V_P
N	1425
$\mathbf{n}_\mu$	9
Confidence level	99%

Source: our treatment methods.



<Fig. 3> The Quantile-Quantile representation of the sample studied on September 14, 2008**.**



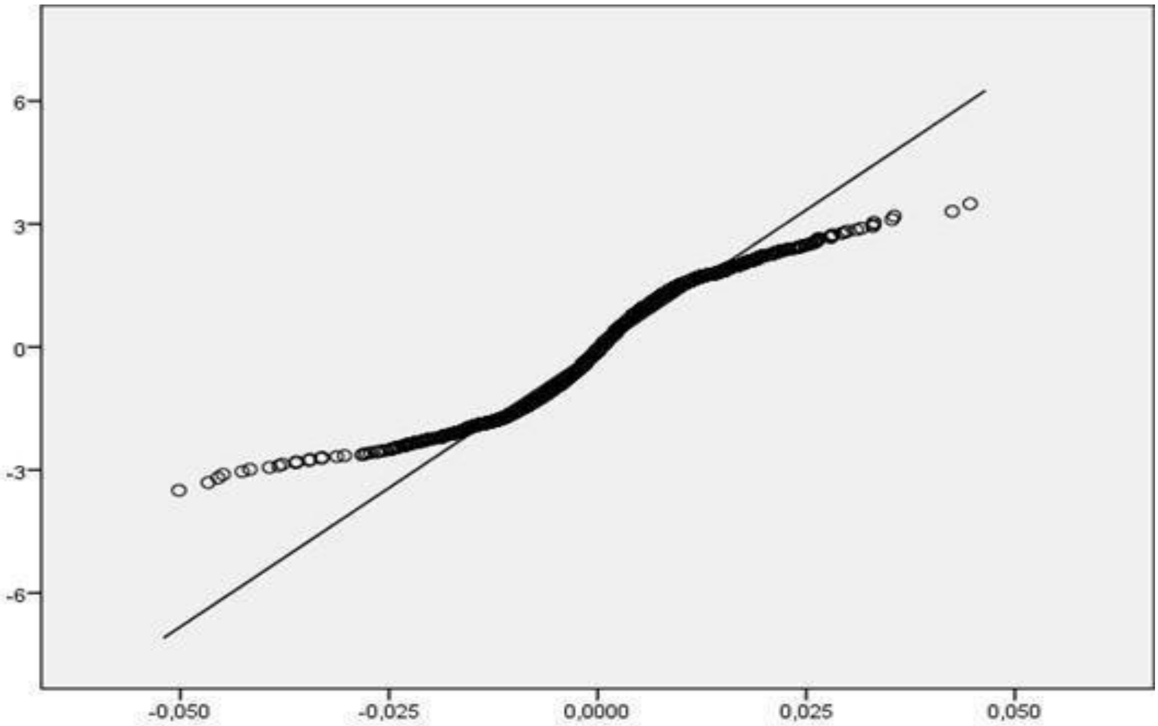
<Fig. 4> Comparison between real losses and estimated losses for the 100 observations after September 14, 2008.

The table 3 and the figure 6 present the results obtained for the March 02, 2020.

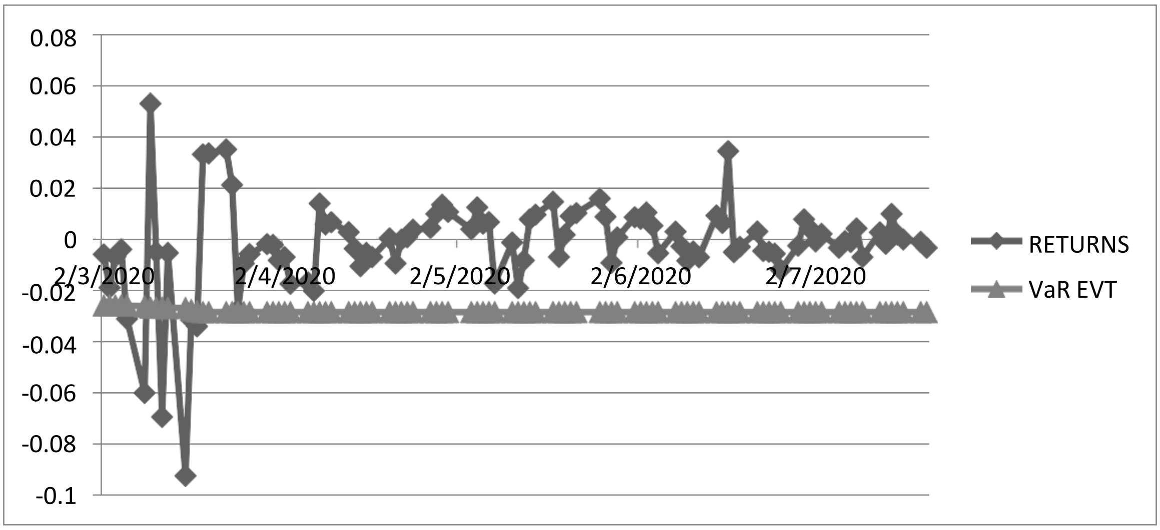
<Table 3> The results of the optimization on Mars 02, 2020.

$\lvert \mu \rvert$	0.02542 V_P
β	0.0000001
ξ	0.09.V_P
N	4254
n_μ	26
Confidence level	99%

Source: our treatment methods.



<Fig. 5> The Quantile-Quantile representation of the sample studied on Mars 02, 2020**.**



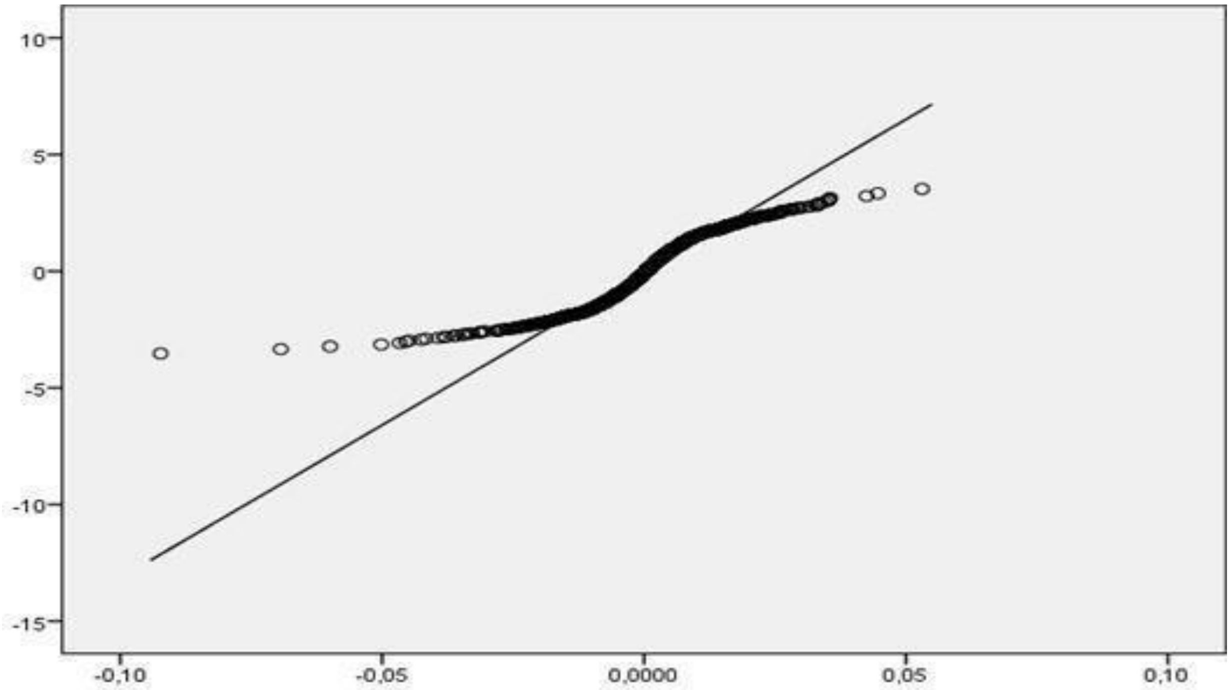
<Fig. 6> Comparison between real losses and estimated losses for the 100 observations after Mars 02, 2020.

The table 4 and the figure 8 present the results obtained for February 24, 2022.

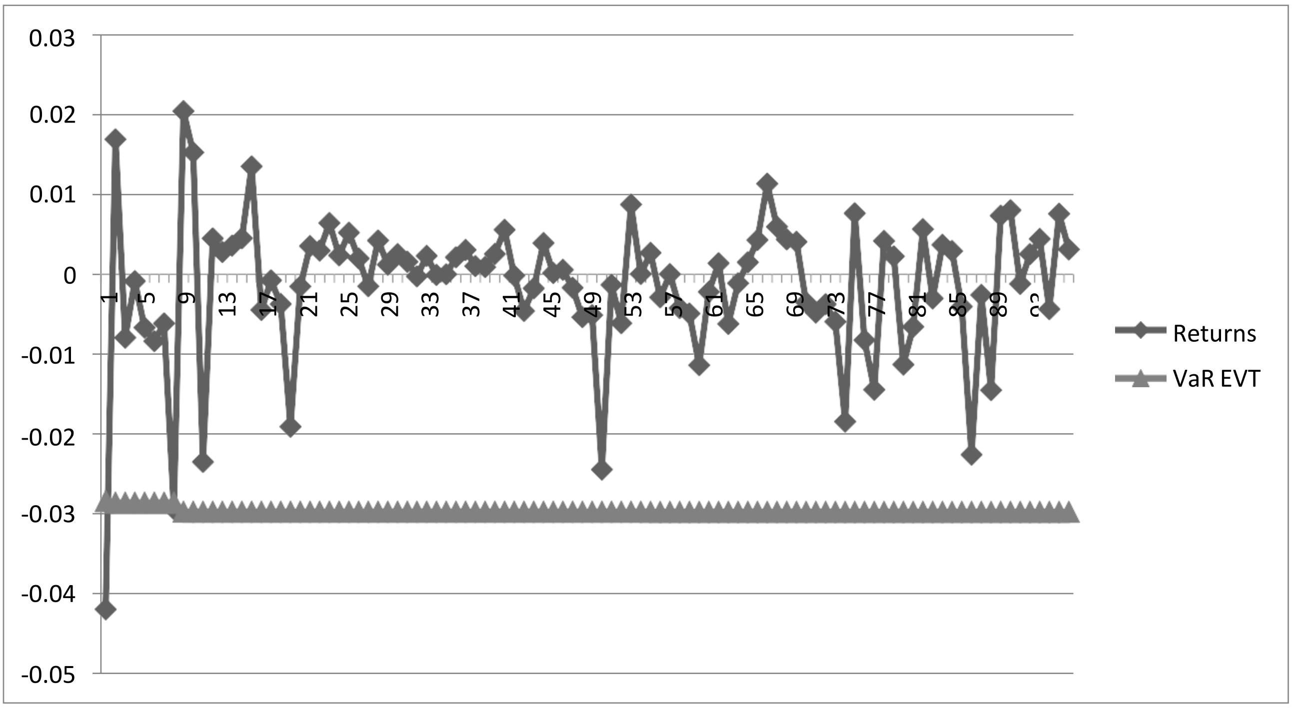
<Table 4> The results of the optimization on February 24, 2022.

$\vert \mu \vert$	0.02805. V_P
β	0.0000001
ξ	0.09. V_P
N	4768
n_μ	27
Confidence level	99%

Source: our treatment methods.



<**Fig. 7**> The Quantile-Quantile representation of the sample studied on February 24, 2022.



<**Fig. 8**> Comparison between real losses and estimated losses for the 100 observations after February 24.

For the three cases, we note that ξ tends to zero so the tail of the distribution tends to have the same properties as those of a normal distribution.

For the first rare event, with a confidence level of 99% and a history containing 89 observations, the real losses exceeded the maximum losses estimated by the EVT VaR six times.

For the second rare event, with a confidence level of 99% and a history containing 1425 observations, the real losses exceeded the maximum losses estimated by the EVT VaR six times.

For the third rare event, with a confidence level of 99% and a history containing 4245 observations, the real losses exceeded the maximum losses estimated by the EVT VaR four times.

For the fourth rare event, with a confidence level of 99% and a history containing 4768 observations, the real losses exceeded the maximum losses estimated by the EVT VaR two times.

Conclusion

The importance of the stock market and its stochastic nature has pushed us to try to apply the theory of the extreme values in this domain. We tried to estimate the maximum losses that should not be exceeded (VaR) during the appearance of some rare event in Morocco and to compare them with the recorded losses. This estimation is very important to forecast the losses during the crises in order to take good decisions and to better manage the unlikely risks.

The Basel standards require for a VaR model, only 1 exceedance per 100 observations. For this reason our application has considered the 100 days following the shock

For the rare event: The Casablanca bombings on 16 May 2003, we have observed during the 100 days following the shock six exceedances for a sample containing 89 observations so we can reject for this date our hypothesis H0: The application of the extreme value theorem using Q-Q plot representation in the estimation of the value at risk is useful in the Moroccan stock market and accept the alternative hypothesis H1 : the application of the extreme value theorem using Q-Q plot representation in the estimation of the value at risk is not useful in the Moroccan stock market.

For the rare event: The banking and financial crisis of 2007-2008, we have observed during the 100 days following the shock six exceedances for a sample containing 1425 observations so we can also reject for this date our hypothesis H0: The application of the extreme value theorem using Q-Q plot representation in the estimation of the value at risk is useful in the Moroccan stock market and accept the alternative hypothesis H1 : the application of the extreme value theorem using Q-Q plot representation in the estimation of the value at risk is not useful in the Moroccan stock market.

For the rare event: The pandemic of covid-19 on 2019-2020, we have observed during the 100 days following the shock four exceedances for a sample containing 4245 observations so we can also reject for this date our hypothesis H0: The application of the extreme value theorem using Q-Q plot representation in the estimation of the value at risk is useful in the Moroccan stock market and accept the alternative hypothesis H1 : the application of the extreme value theorem using Q-Q plot representation in the estimation of the value at risk is not useful in the Moroccan stock market.

Finally, For the rare event: The Russian-Ukrainian conflict on 2022, we have observed two exceedances for a sample containing 4867 observations so we can also reject for this date our hypothesis H0: The application of the extreme value theorem using Q-Q plot representation in the estimation of the value at risk is useful in the Moroccan stock market and accept the alternative hypothesis H1 : the application of the extreme value theorem using Q-Q plot representation in the estimation of the value at risk is not useful in the Moroccan stock market.

We conclude that the efficacy of the VaR estimated by the extreme value theory at the Moroccan stock market depends on the size of the sample. To be applied, this theory must have an interesting history. The complexity of this method consists in the choice of the levelμ. In this work we have chosen this level by the QQ-PLOT method but there are other more precise methods that we wish to apply soon.

References

Desheng Wu, Pandemic Risk management in operations and finance , modeling the impact of covid-19,2020.
Yassine bakkar (2018). Contributions of the extreme values theory in the calculation of value-at-risk : 10–13.
De haan L.,Ferreira A. (2006) Extreme value theory : an introduction
Hopkin (2018) Paul,Fundamentals of risk management understanding, evaluating and implementing effective risk management.
John Wiley, Sons Ltd(2008). Market risk analysis.
Michael B.Miller(2014).Mathematics & Statistics for financial Risk Management 2ème édition.
Moorad choudhry,Carol Alexander (2013) .An introduction to Value-at-Risk edition 5.
Olivier hassid(2008). Risk management 2nd edition.
Philippe jorion(2006).Value at risk : the new benchmark for managing financial risk edition 3.
Samuel kotz, Saralees Nadarajah (2000) .Extreme value distributions theory an applications.
Samuel kotz, Saralees Nadarajah (2001) .Extreme value distributions theory an applications.
Wiley (2016).Extreme events in finance : A handbook of extreme value theory and its applications.
Michael B.Miller, Mathematics & Statistics for financial Risk Management 2ème édition,2014
Olivier hassid, Risk management 2nd edition ,2008.
John Wiley & Sons Ltd , Market risk analysis, 2008.
Harry M.Markowitz , Portfolio selection : efficient diversification of investments , 1968.
E.A Cornish , R.A. Fisher, Moments and cumulants in the specification of distributions,revue de l'institut international de statistiques , 1938
Moorad choudhry,Carol Alexander, An introduction to Value-at-Risk edition 5,2013.
Philippe jorion , Value at risk : the new benchmark for managing financial risk edition 3,2006.
Hopkin, Paul,Fundamentals of risk management understanding, evaluating and implementing effective risk management , 2018.
Paolo Brandimarte, Handbook in Monte Carlo simulation:Applications in financial engineering, risk management and economics , 2014.
Marek capinski,Tomas zatawniah , Mathematics for finance: An introduction to financial engineering, 2003.
Robert kosowski,Salih N.Neftci , Principles of financial Engineering , 2015.
Dan stefanica , Aprimer for the mathematics of financial engineering , 2008
David Ruppert,David s.Matteson : Statitstics and Data Analysis for financial engineering : with R examples,2015
Perry H.beaumount , Financial Engineering principles : A unified theory for financial product analysis and valuation,2004

[1] Laszlo gyorfi,Gyorgy Ottucsak, Harro walk, Machine learning for financial engineering ,2012
John D.Finnerty, Project financing : Asset-based financial engineering, 2013
Niall fraser, Elizabeth jewkes,Engineering economics financial decision making for engineers, 2012
Dan stefanica ,Solutions Manual A primer for the mathematics of financial engineering,2008
Robert Dubil, Financial Engineering and arbitrage in financial markets , 2011.
Rupak chatterjee , Practical methods of financial engineering and risk management : Tools for modern financial professionals.
Bruno remillard , Dtatistical methods for financial engineering , 2013
Tanya S.beder , Cara M.Marshall ,Financial engineering : The evolution of a profession, 2011.
Salih N.Neftci ,Principles of financial engineering, Second edition, 2008.
Abol Ardalan , Economic and financial analysis for engineering and project management , 1999